Digesto para la programación imperativa

Algoritmos y Estructuras de Datos I

1. **Relación weakest precondition y terna de Hoare:**

$[wp.S.Q = P] \iff \begin{cases} (i)\ \{P\}\ S\ \{Q\} \\ (ii)\ \{P_0\}\ S\ \{Q\} \Rightarrow [P_0 \Rightarrow P] \end{cases}$	$\{P\}\ S\ \{Q\} \equiv [P \Rightarrow wp.S.Q]$

2. **Teorema fundamental de la computación (asignación):**

Verificación con terna de Hoare:	Weakest precondition:	Programa anotado:
$\{P\}$ $x := E \equiv [P \Rightarrow Q(x := E)]$ $\{Q\}$	$[wp.(x := E).Q \equiv Q(x := E)]$	$\{Q(x := E)\}$ $x := E$ $\{Q\}$

Si no se asumen bien definidas las expresiones E:

Verificación con terna de Hoare:	Weakest precondition:
$\{P\}$ $x := E \equiv [P \Rightarrow def.E \wedge Q(x := E)]$ $\{Q\}$	$[wp.(x := E).Q \equiv Q(x := E) \wedge Def.E]$
Programa anotado: $\{Q(x := E) \wedge Def.E\}$ $x := E$ $\{Q\}$	

3. **Skip:**

Verificación con terna de Hoare:	Weakest precondition:	Programa anotado:
$\{P\}\ skip\ \{Q\} \equiv [P \Rightarrow Q]$	$[wp.skip.Q \equiv Q]$	$\{Q\}$ $skip$ $\{Q\}$

4. **Abort:**

Verificación con terna de Hoare:	Weakest precondition:	Programa anotado:
$\{P\}\ abort\ \{Q\} \equiv [P \equiv False]$	$[wp.abort.Q \equiv False]$	$\{False\}$ $abort$ $\{Q\}$

5. **Composición o concatenación de sentencias:**

Verificación con terna de Hoare:	Weakest precondition:
$\{P\}\ S;T\ \{Q\} \equiv$ Existe R tal que $\{P\}\ S\ \{R\} \wedge \{R\}\ T\ \{Q\}$	$[wp.S;T.Q \equiv wp.S.(wp.T.Q)]$
Programa anotado: $\begin{matrix}\{P\}\\ S\\ \{R\}\\ T\\ \{Q\}\end{matrix} \equiv \begin{matrix}\{P\}\\ S\\ \{R\}\end{matrix} \wedge \begin{matrix}\{R\}\\ T\\ \{Q\}\end{matrix}$ **o** $\begin{matrix}\{wp.S.(wp.T.Q)\}\\ S\\ \{wp.T.Q\}\\ T\\ \{Q\}\end{matrix}$	

6. **Alternativa (if):**

Verificación con ternas de Hoare:

$$\begin{array}{llll} \{P\} & \textbf{if} \;\; B_0 \to S_0 & \{Q\} \;\; \equiv & [P \Rightarrow (B_0 \vee B_1 \vee \ldots \vee B_n)] \\ & \Box \;\; B_1 \to S_1 & & \wedge\{P \wedge B_0\}\ S_0\ \{Q\} \\ & \vdots & & \wedge\{P \wedge B_1\}\ S_1\ \{Q\} \\ & \Box \;\; B_n \to S_n & & \vdots \\ & \textbf{fi} & & \wedge\{P \wedge B_n\}\ S_n\ \{Q\} \end{array}$$

Weakest precondition:

$$wp.if.Q \equiv [(B_0 \vee B_1 \vee \ldots \vee B_n) \wedge (B_0 \Rightarrow wp.S_0.Q) \wedge \ldots \wedge (B_n \Rightarrow wp.S_n.Q)]$$

Programa anotado:

$$\begin{array}{l} \{P \wedge (B_0 \vee B_1 \vee \ldots \vee B_n)\} \\ \;\textbf{if}\ B_0 \to \\ \quad \{B_0 \wedge P\} \\ \quad S_0 \\ \quad \{Q\} \\ \;\Box\ B_1 \to \\ \quad \{B_1 \wedge P\} \\ \quad S_1 \\ \quad \{Q\} \\ \;\vdots \\ \;\Box\ B_n \to \\ \quad \{B_n \wedge P\} \\ \quad S_n \\ \quad \{Q\} \\ \;\textbf{fi} \\ \{Q\} \end{array}$$

o

$$\begin{array}{l} \{(B_0 \vee B_1 \vee \ldots \vee B_n) \wedge \\ \;(B_0 \Rightarrow wp.S_0.Q) \wedge \ldots \wedge (B_n \Rightarrow wp.S_n.Q)\} \\ \;\textbf{if}\ B_0 \to \\ \quad \{B_0 \wedge wp.S_0.Q\} \\ \quad S_0 \\ \quad \{Q\} \\ \;\Box\ B_1 \to \\ \quad \{B_1 \wedge wp.S_1.Q\} \\ \quad S_1 \\ \quad \{Q\} \\ \;\vdots \\ \;\Box\ B_n \to \\ \quad \{B_n \wedge wp.S_n.Q\} \\ \quad S_n \\ \quad \{Q\} \\ \;\textbf{fi} \\ \{Q\} \end{array}$$

7. **Teorema de invariancia:**

Verificación con ternas de Hoare:

$$\begin{array}{llll} \{P\} & \textbf{do}\ B_0 \to S_0 & \{Q\} \;\; \equiv & \text{Existe } I \text{ (invariante) tal que} \\ & \Box \;\; B_1 \to S_1 & & [P \Rightarrow I] \\ & \vdots & & \wedge[I \wedge \neg B_0 \wedge \neg B_1 \wedge \ldots \wedge \neg B_n \Rightarrow Q] \\ & & & \wedge\{I \wedge B_i\}\ S_i\ \{I\} \quad 0 \leq i \leq n \\ & \Box \;\; B_n \to S_n & & \wedge \ \text{Existe } t : Estados \mapsto Int \text{ tal que} \\ & \textbf{od} & & \quad \text{(i) } [I \wedge (B_0 \vee B_1 \vee \ldots \vee B_n) \ \Rightarrow \ t \geq 0] \\ & & & \quad \text{(ii) } \{I \wedge B_i \wedge t = T\}\ S_i\ \{t < T\} \quad 0 \leq i \leq n \end{array}$$

Programa anotado:

$$\begin{array}{l} \quad \{I\} \\ \textbf{do}\ B_0 \to \\ \quad \{I \wedge B_0 \wedge t = T\} \\ \quad S_0 \\ \quad \{I \wedge t < T\} \\ \Box\ B_1 \to \\ \quad \{I \wedge B_1 \wedge t = T\} \\ \quad S_1 \\ \quad \{I \wedge t < T\} \\ \vdots \\ \Box\ B_n \to \\ \quad \{I \wedge B_n \wedge t = T\} \\ \quad S_n \\ \quad \{I \wedge t < T\} \\ \textbf{od} \\ \{I \wedge \neg B_0 \wedge \neg B_1 \wedge \ldots \wedge \neg B_n\} \end{array}$$

y $[I \wedge (B_0 \vee B_1 \vee \ldots \vee B_n) \ \Rightarrow \ t \geq 0]$

8. **Otras anotaciones**

$\begin{array}{c}\{R\}\\ \{P\}\\ S\\ \{Q\}\end{array} \quad \equiv \quad [R \Rightarrow P] \;\wedge\; \begin{array}{c}\{P\}\\ S\\ \{Q\}\end{array}$	$\begin{array}{c}\{P\}\\ S\\ \{Q\}\\ \{R\}\end{array} \quad \equiv \quad \begin{array}{c}\{P\}\\ S\\ \{Q\}\end{array} \;\wedge\; [Q \Rightarrow R]$

9. **Propiedades**

- $\{P\}\ S\ \{False\} \;\equiv\; [P \equiv False]$ (Exclusión de milagros)
- $[wp.S.False \equiv False]$
- $[wp.S.Q \wedge wp.S.R \;\equiv\; wp.S.(Q \wedge R)]$
- $[wp.S.Q \vee wp.S.R \;\Rightarrow\; wp.S.(Q \vee R)]$