

Digesto para la programación imperativa

Algoritmos y Estructuras de Datos I

1. Skip:

Verificación con terna de Hoare: $\{P\} \text{ skip } \{Q\} \equiv [P \Rightarrow Q]$	Weakest precondition: $[wp.\text{skip}.Q \equiv Q]$	Programa anotado: $\{Q\}$ <i>skip</i> $\{Q\}$
--	--	--

2. Abort:

Verificación con terna de Hoare: $\{P\} \text{ abort } \{Q\} \equiv [P \equiv \text{False}]$	Weakest precondition: $[wp.\text{abort}.Q \equiv \text{False}]$	Programa anotado: $\{\text{False}\}$ <i>abort</i> $\{Q\}$
---	--	--

3. Asignación:

Verificación con terna de Hoare: $\{P\}$ $x := E \equiv [P \Rightarrow Q(x := E)]$ $\{Q\}$	Weakest precondition: $[wp.(x := E).Q \equiv Q(x := E)]$	Programa anotado: $\{Q(x := E)\}$ $x := E$ $\{Q\}$
---	---	---

Si no se asumen bien definidas las expresiones E :

Verificación con terna de Hoare: $\{P\}$ $x := E \equiv [P \Rightarrow \text{def}.E \wedge Q(x := E)]$ $\{Q\}$	Weakest precondition: $[wp.(x := E).Q \equiv Q(x := E) \wedge \text{Def}.E]$
Programa anotado: $\{Q(x := E) \wedge \text{Def}.E\}$ $x := E$ $\{Q\}$	

4. Composición o concatenación de sentencias:

Verificación con terna de Hoare: $\{P\} S; T \{Q\} \equiv \text{Existe } R \text{ tal que } \{P\} S \{R\} \wedge \{R\} T \{Q\}$	Weakest precondition: $[wp.S; T.Q \equiv wp.S.(wp.T.Q)]$
Programa anotado: $\begin{array}{c} \{P\} \\ S \\ \{R\} \\ T \\ \{Q\} \end{array} \equiv \begin{array}{c} \{P\} \\ S \\ \{R\} \end{array} \wedge \begin{array}{c} \{R\} \\ T \\ \{Q\} \end{array} \quad \circ \quad \begin{array}{c} \{wp.S.(wp.T.Q)\} \\ S \\ \{wp.T.Q\} \\ T \\ \{Q\} \end{array}$	

5. Alternativa (if):

Verificación con ternas de Hoare:	
$\{P\} \text{ if } B_0 \rightarrow S_0 \quad \{Q\} \equiv [P \Rightarrow (B_0 \vee B_1 \vee \dots \vee B_n)]$ $\square B_1 \rightarrow S_1 \quad \wedge \{P \wedge B_0\} S_0 \{Q\}$ $\vdots \quad \wedge \{P \wedge B_1\} S_1 \{Q\}$ $\square B_n \rightarrow S_n \quad \vdots$ $\text{fi} \quad \wedge \{P \wedge B_n\} S_n \{Q\}$	
Weakest precondition:	
$wp.if.Q \equiv [(B_0 \vee B_1 \vee \dots \vee B_n) \wedge (B_0 \Rightarrow wp.S_0.Q) \wedge \dots \wedge (B_n \Rightarrow wp.S_n.Q)]$	
Programa anotado:	
$\{P \wedge (B_0 \vee B_1 \vee \dots \vee B_n)\}$ $\text{if } B_0 \rightarrow$ $\quad \{B_0 \wedge P\}$ $\quad S_0$ $\quad \{Q\}$ $\square B_1 \rightarrow$ $\quad \{B_1 \wedge P\}$ $\quad S_1$ $\quad \{Q\}$ \vdots $\square B_n \rightarrow$ $\quad \{B_n \wedge P\}$ $\quad S_n$ $\quad \{Q\}$ fi $\{Q\}$	$\{(B_0 \vee B_1 \vee \dots \vee B_n) \wedge$ $(B_0 \Rightarrow wp.S_0.Q) \wedge \dots \wedge (B_n \Rightarrow wp.S_n.Q)\}$ $\text{if } B_0 \rightarrow$ $\quad \{B_0 \wedge wp.S_0.Q\}$ $\quad S_0$ $\quad \{Q\}$ $\square B_1 \rightarrow$ $\quad \{B_1 \wedge wp.S_1.Q\}$ $\quad S_1$ $\quad \{Q\}$ o \vdots $\square B_n \rightarrow$ $\quad \{B_n \wedge wp.S_n.Q\}$ $\quad S_n$ $\quad \{Q\}$ fi $\{Q\}$

6. Teorema de invariancia:

Verificación con ternas de Hoare:	
$\{P\} \equiv \text{Existe } I \text{ (invariante) tal que}$ $\text{do } B \rightarrow \quad [P \Rightarrow I]$ $\quad S \quad \wedge [I \wedge \neg B \Rightarrow Q]$ $\text{od} \quad \wedge \{I \wedge B\} S \{I\}$ $\{Q\} \quad \wedge \text{Existe } t : \text{Estados} \mapsto \text{Int} \text{ tal que}$ $\quad \text{(i) } [I \wedge B \Rightarrow t \geq 0]$ $\quad \text{(ii) } \{I \wedge B \wedge t = T\} S \{t < T\}$	
Programa anotado:	
$\{I\}$ $\text{do } B \rightarrow$ $\quad \{I \wedge B\}$ $\quad S$ $\quad \{I\}$ od $\{I \wedge \neg B\}$	$[I \wedge B \Rightarrow t \geq 0]$ $\text{do } B \rightarrow$ $\quad \{I \wedge B \wedge t = T\}$ $\quad S$ $\quad \{t < T\}$ od

7. Anotaciones Secuenciales

$\begin{array}{c} \{R\} \\ \{P\} \\ S \\ \{Q\} \end{array} \equiv [R \Rightarrow P] \wedge \begin{array}{c} \{P\} \\ S \\ \{Q\} \end{array}$	$\begin{array}{c} \{P\} \\ S \\ \{Q\} \\ \{R\} \end{array} \equiv \begin{array}{c} \{P\} \\ S \\ \{Q\} \end{array} \wedge [Q \Rightarrow R]$
--	--

8. Relación weakest precondition y terna de Hoare:

$[wp.S.Q = P] \iff \left\{ \begin{array}{l} (i) \{P\} S \{Q\} \\ (ii) \{P_0\} S \{Q\} \Rightarrow [P_0 \Rightarrow P] \end{array} \right.$	$\{P\} S \{Q\} \equiv [P \Rightarrow wp.S.Q]$
--	---

9. Propiedades

- $\{P\} S \{False\} \equiv [P \equiv False]$ (Exclusión de milagros)
- $[wp.S.False \equiv False]$
- $[wp.S.Q \wedge wp.S.R \equiv wp.S.(Q \wedge R)]$
- $[wp.S.Q \vee wp.S.R \Rightarrow wp.S.(Q \vee R)]$