

Algoritmos y Estructuras de Datos II

Más sobre backtracking

3 de junio de 2015

Clase de hoy

- 1 Repaso
 - Repaso algoritmos avanzados
- 2 Backtracking = DFS sobre un grafo implícito
- 3 Ocho reinas

Repaso general

- cómo vs. qué
- 3 partes
 - 1 análisis de algoritmos
 - 2 tipos de datos
 - 3 técnicas de resolución de problemas
 - divide y vencerás
 - algoritmos voraces
 - **backtracking**
 - programación dinámica: problema de la moneda, problema de la mochila, algoritmo de Floyd
 - **recorrida de grafos**

Clase de hoy

- 1 Repaso
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- 3 Ocho reinas

Técnicas de resolución de problemas

- Algoritmos voraces
 - Cuando tenemos un criterio de selección que garantiza optimalidad
- Backtracking
 - Cuando no tenemos un criterio así
 - solución top-down
 - en general es exponencial
 - hoy veremos un nuevo problema: 8 reinas
- Programación dinámica
 - construye una tabla bottom-up
 - evita repetir cálculos
 - pero realiza algunos cálculos inútiles.
- DFS y BFS
 - estrategias para recorrer grafos
 - hoy veremos: backtracking = DFS sobre un grafo implícito

Problema de la moneda

Primera solución que usa backtracking

Recordemos la primera solución al problema de la moneda usando backtracking:

$$m(i, j) = \begin{cases} 0 & j = 0 \\ \infty & j > 0 \wedge i = 0 \\ m(i - 1, j) & d_i > j > 0 \wedge i > 0 \\ \min(m(i - 1, j), 1 + m(i, j - d_i)) & j \geq d_i > 0 \wedge i > 0 \end{cases}$$

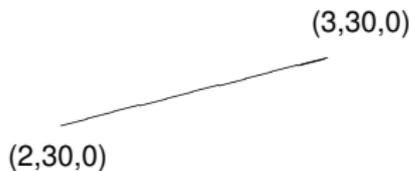
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 30$

(3,30,0)

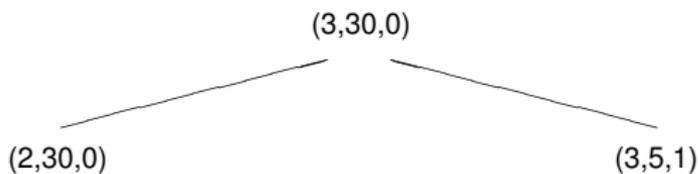
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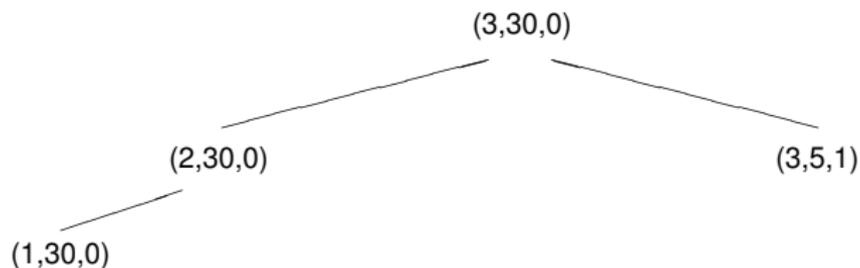
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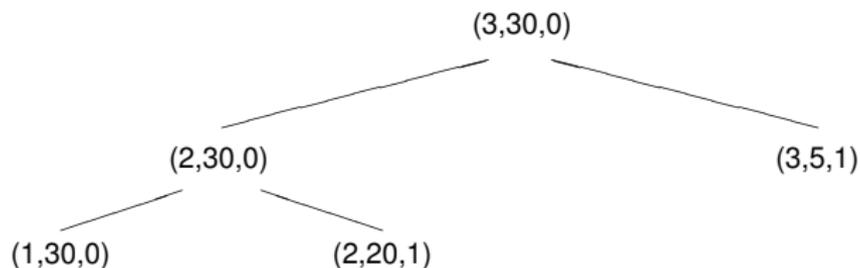
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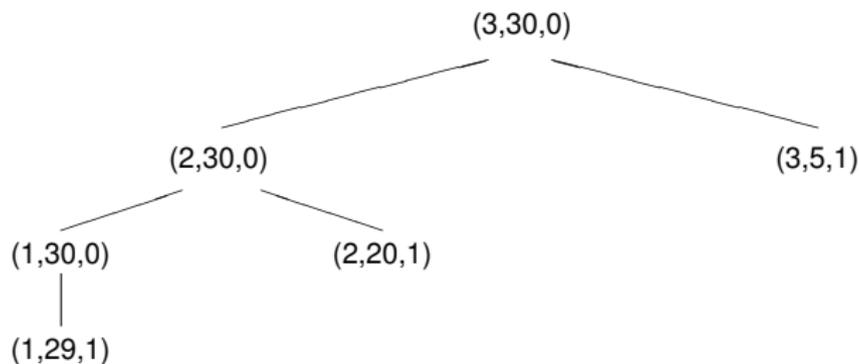
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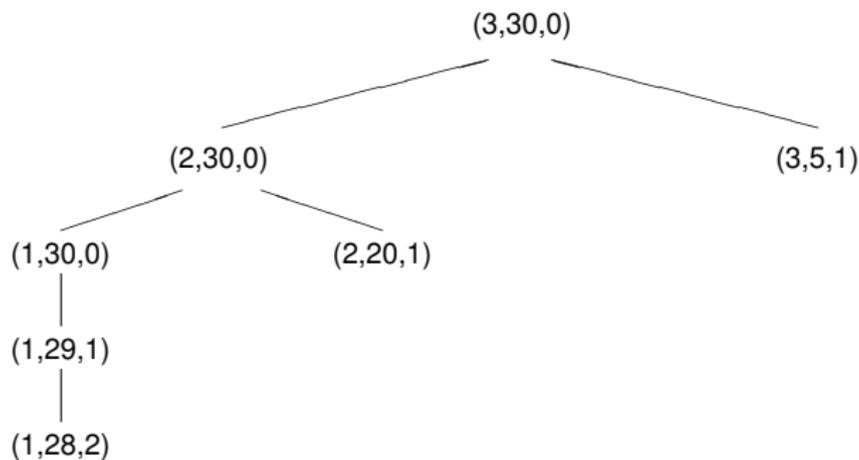
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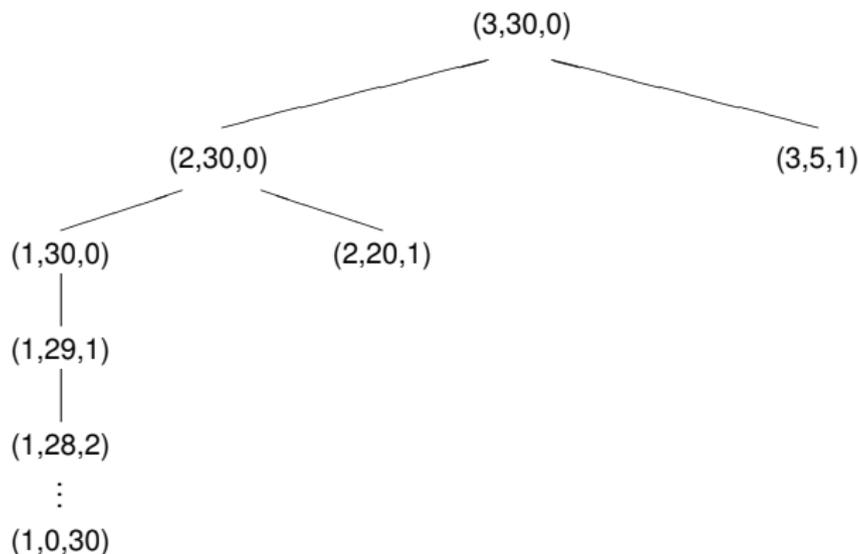
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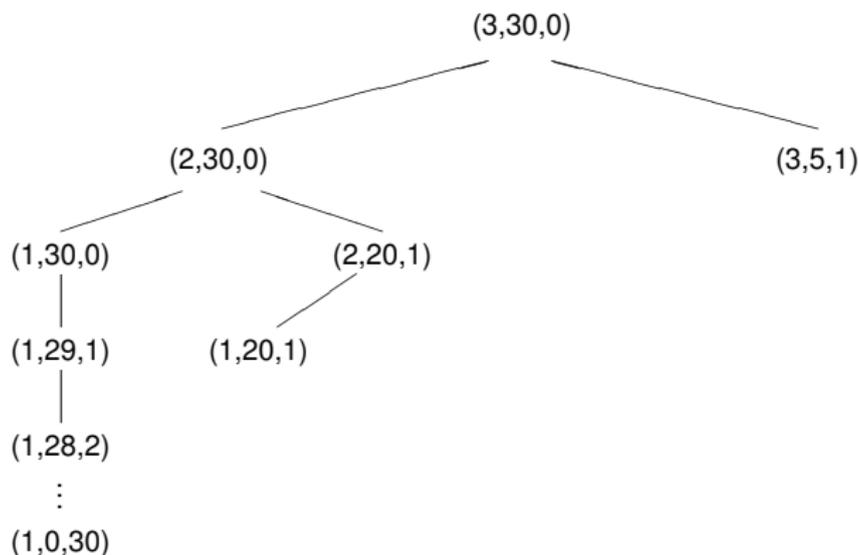
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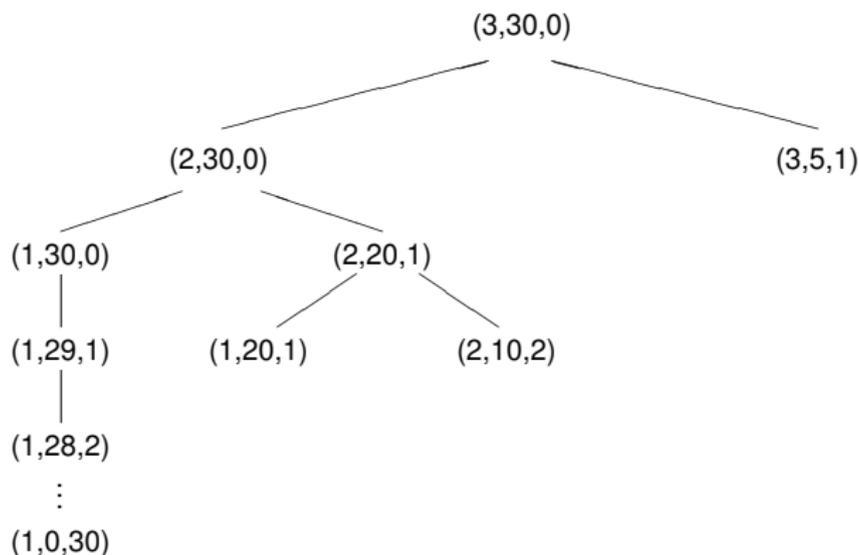
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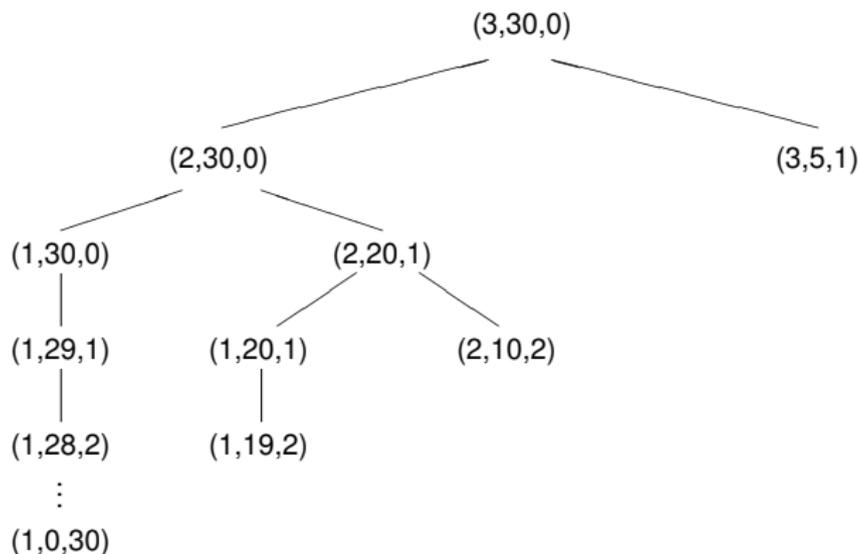
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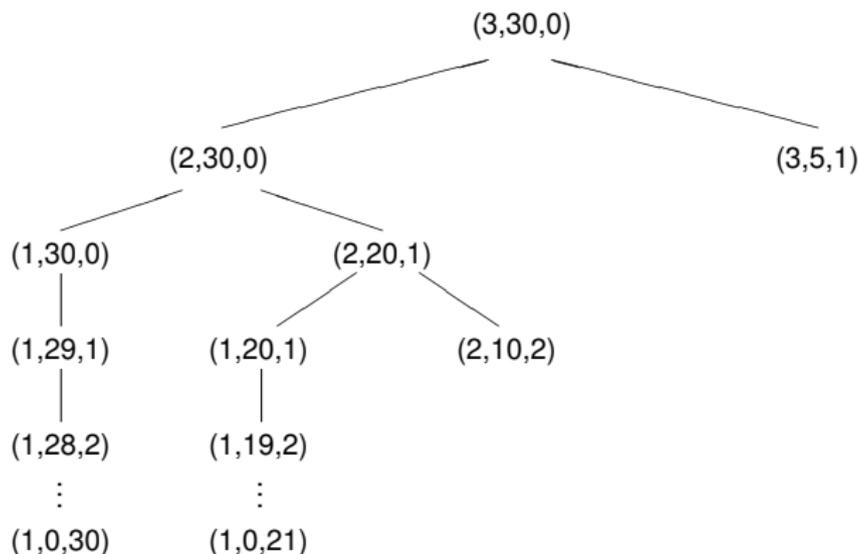
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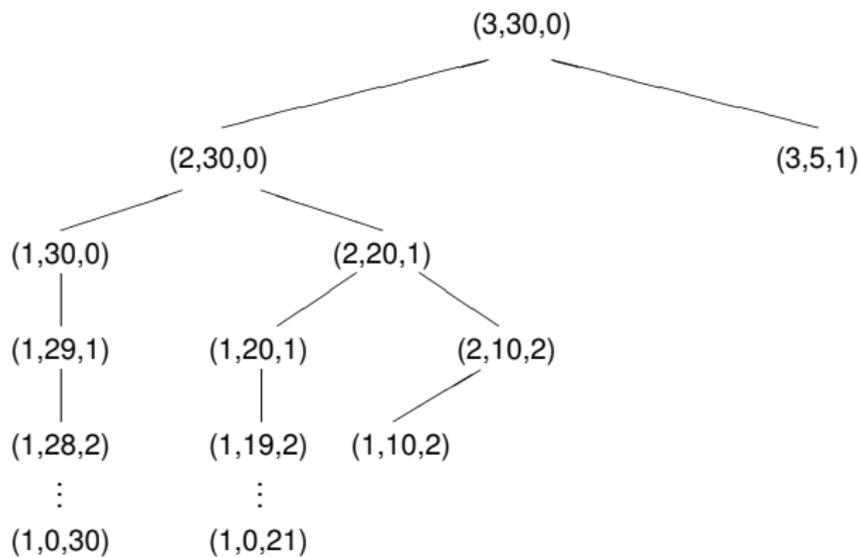
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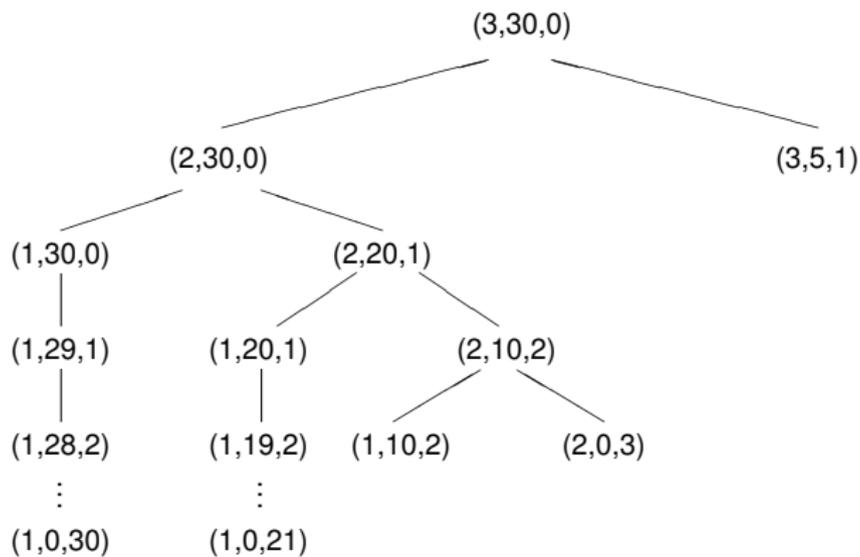
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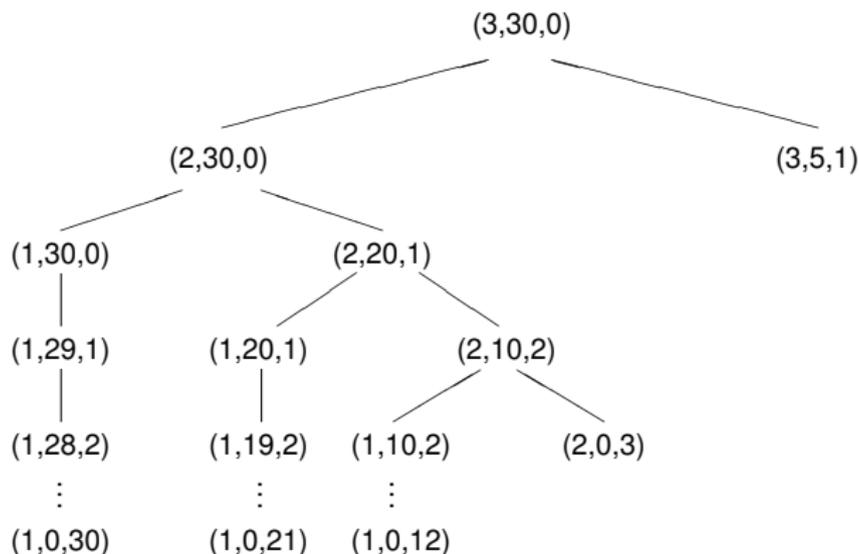
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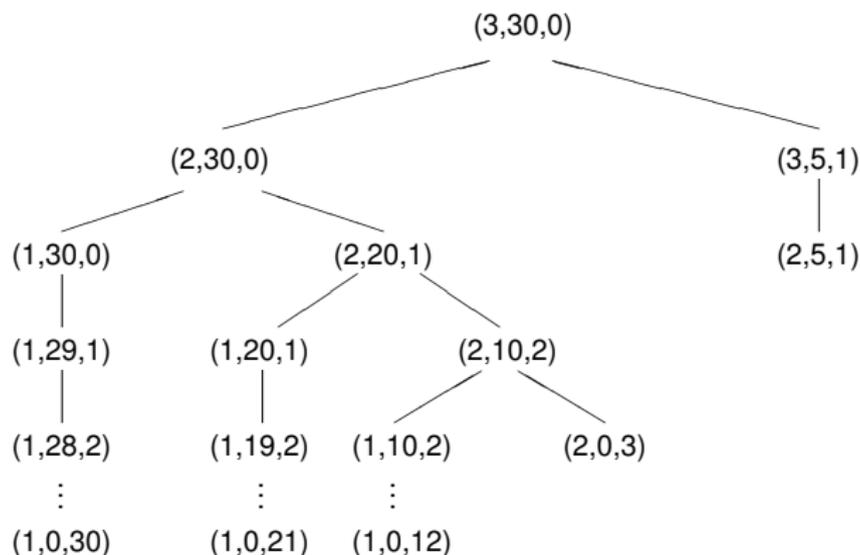
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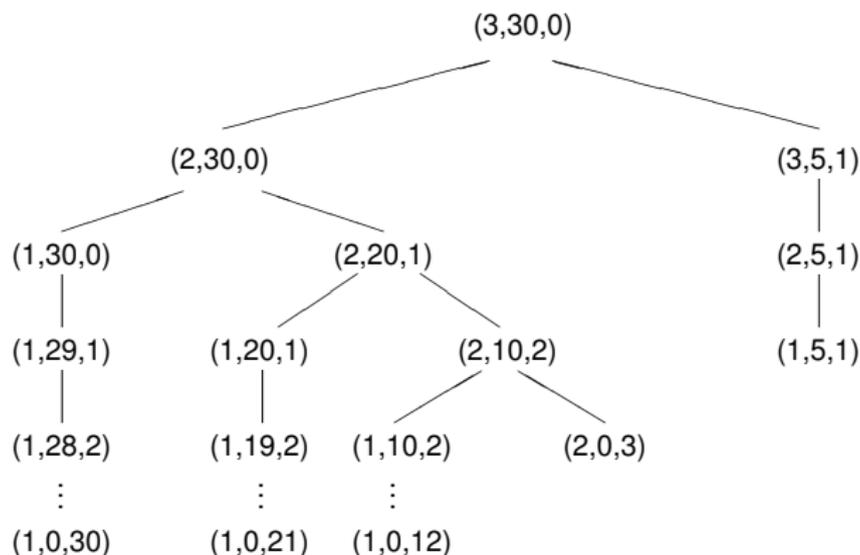
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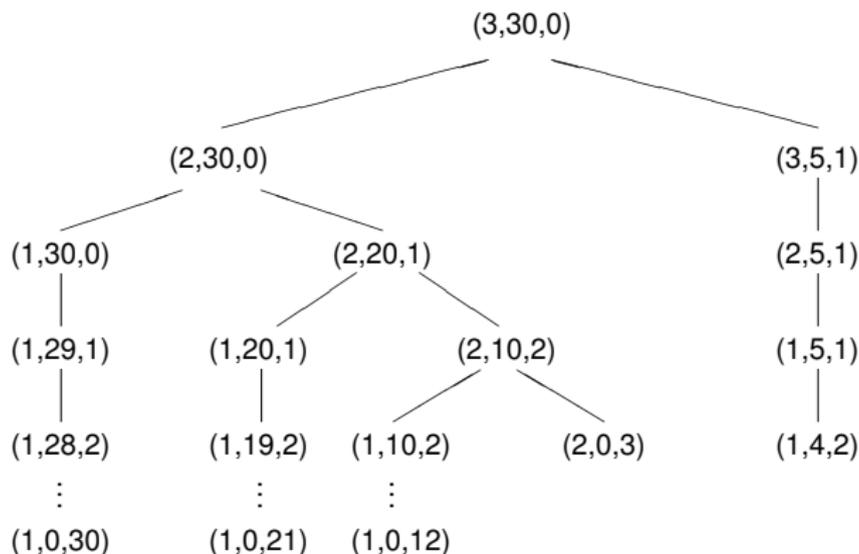
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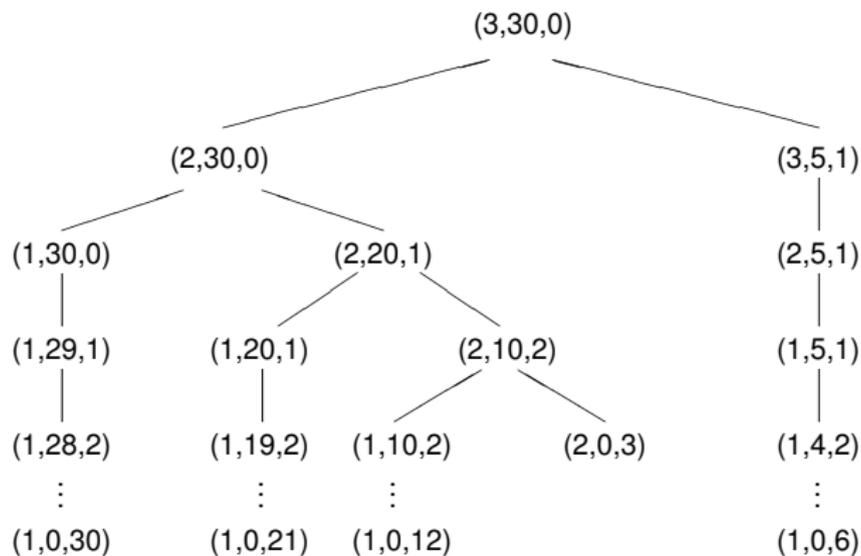
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Grafo implícito

Definición general

- Desde el vértice (i, j, x) , si $i, j > 0$ y $d_i < j$ existe una única arista a al vértice $(i - 1, j, x)$.
- En cambio si $j \leq d_i$ existen dos aristas:
 - una a $(i - 1, j, x)$
 - y otra a $(i, j - d_i, x + 1)$.
- la raíz es el vértice $(n, k, 0)$.

Problema de la moneda

Segunda solución que usa backtracking

Recordemos otra solución al problema de la moneda usando backtracking:

$$m(i, j) = \begin{cases} 0 & j = 0 \\ 1 + \min(\{m(i', j - d_{i'}) \mid 1 \leq i' \leq i \wedge d_{i'} \leq j\}) & j > 0 \end{cases}$$

Grafo implícito

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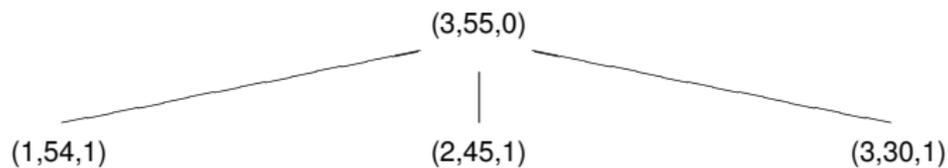
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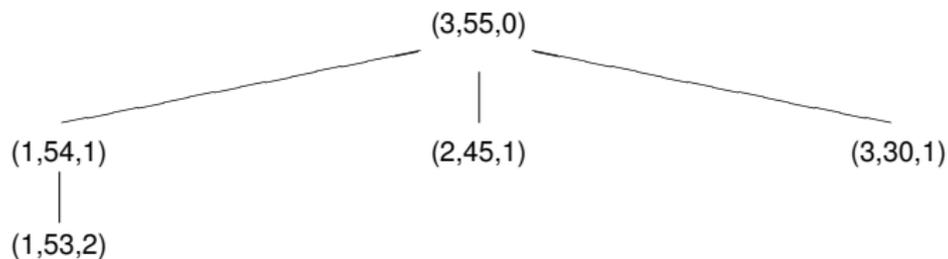
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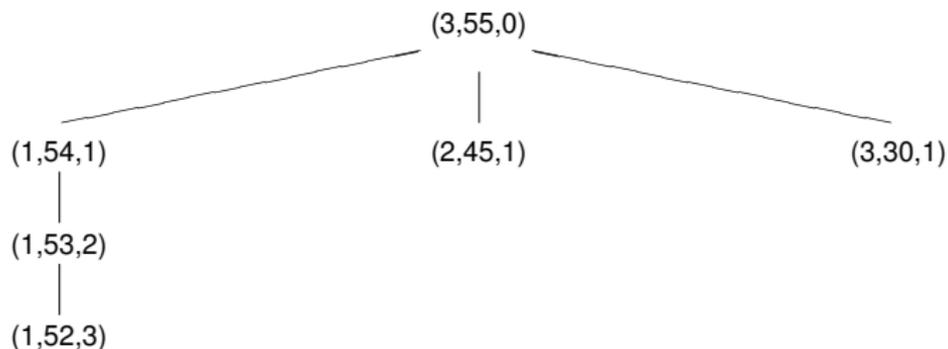
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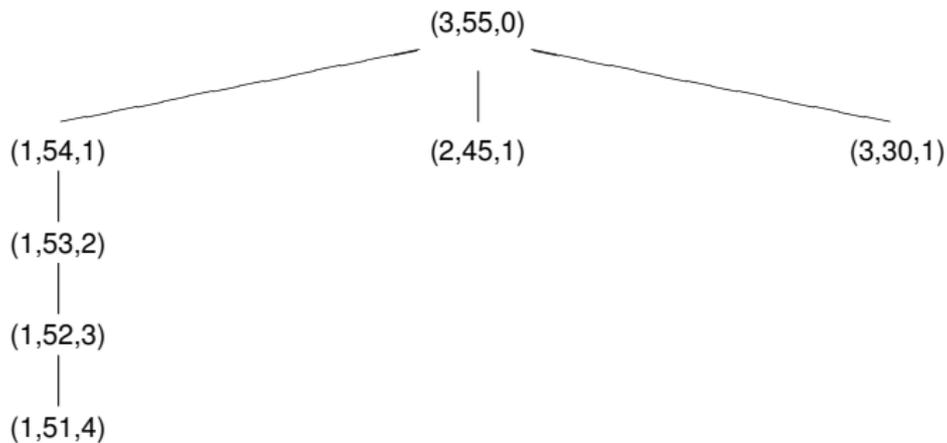
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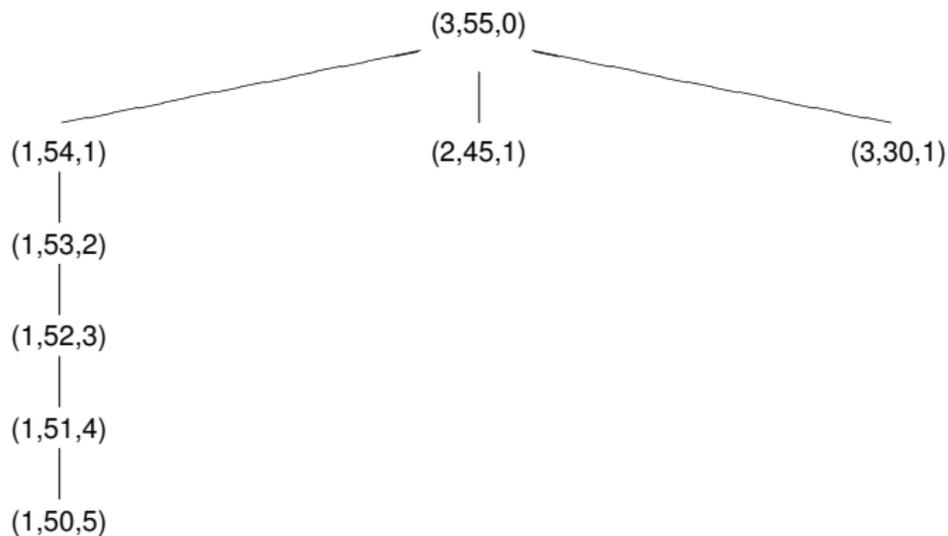
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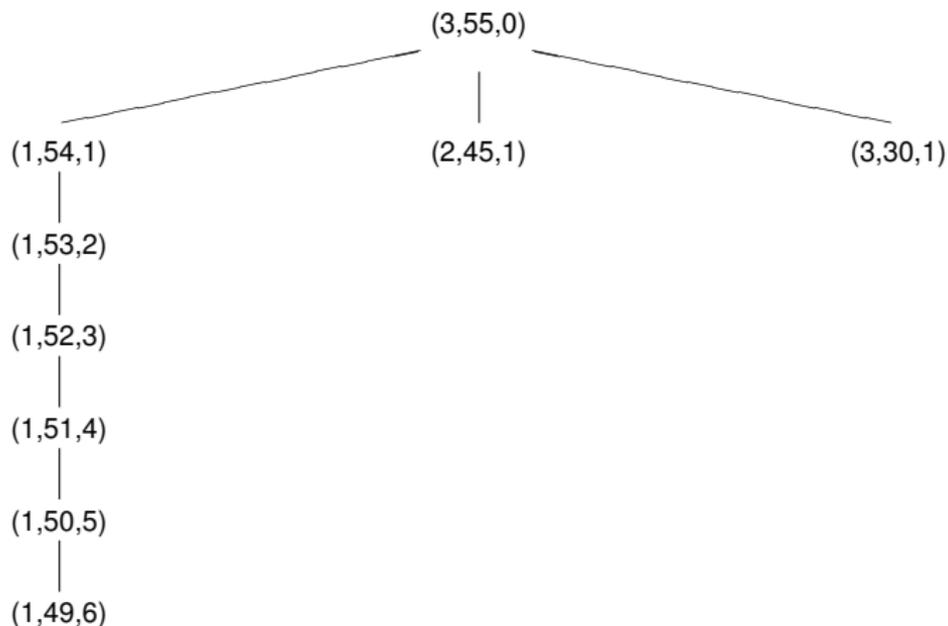
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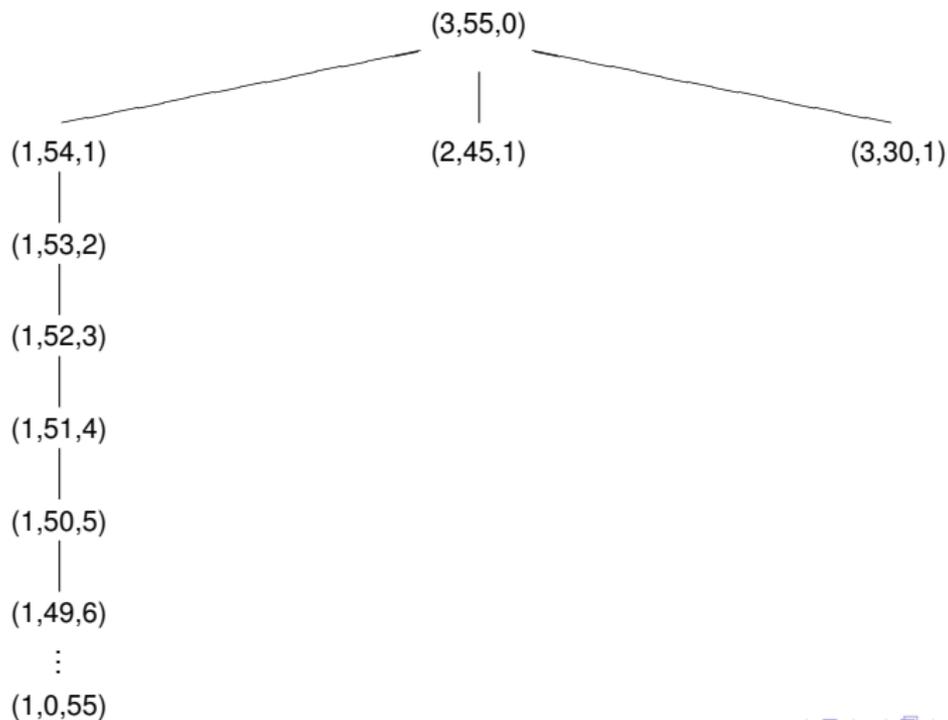
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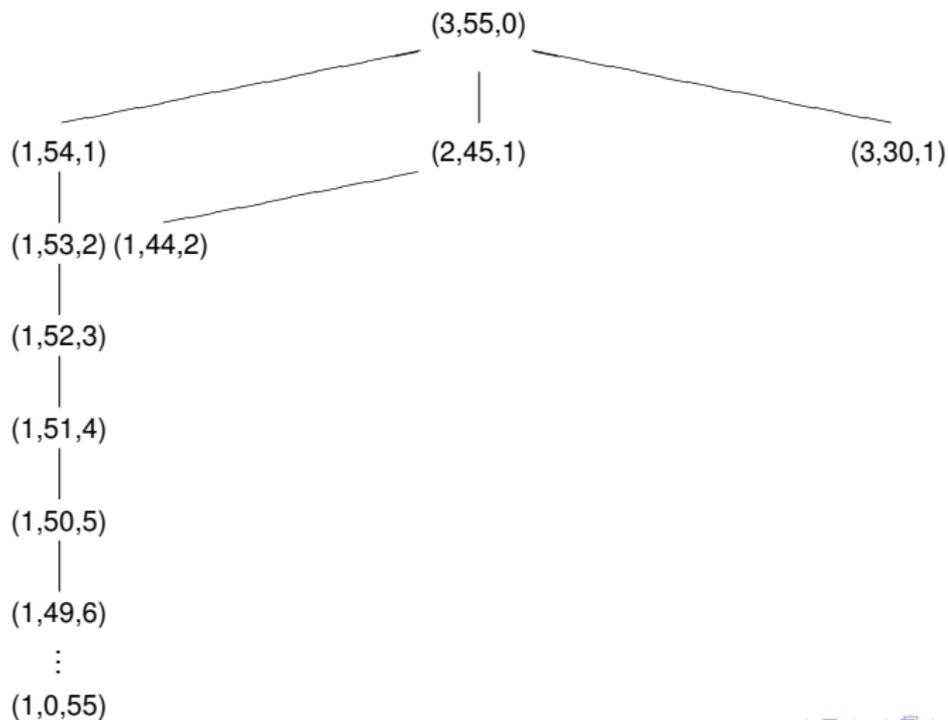
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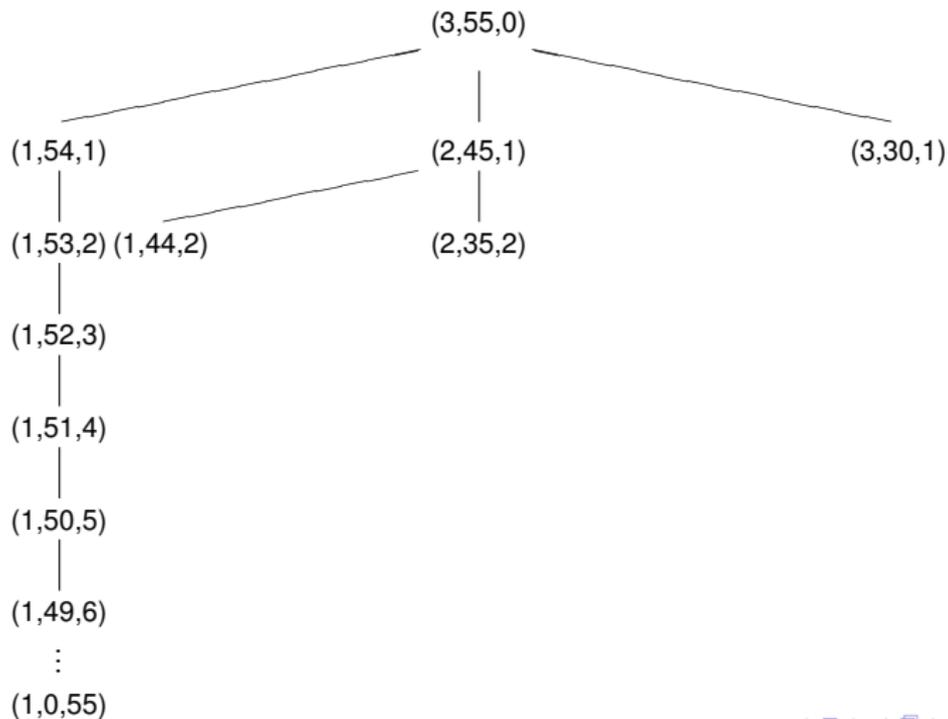
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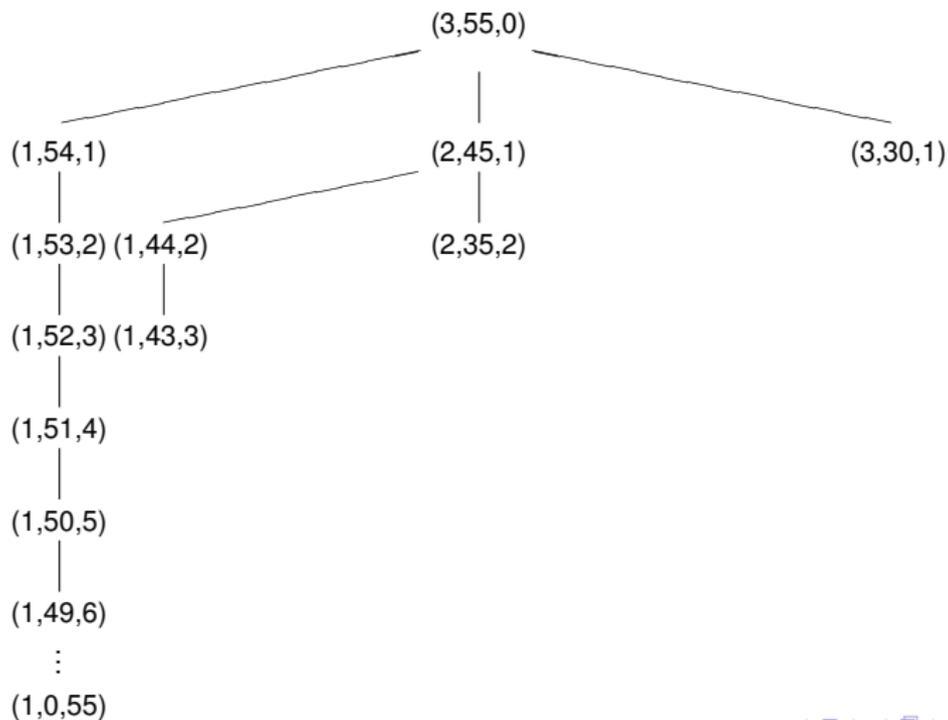
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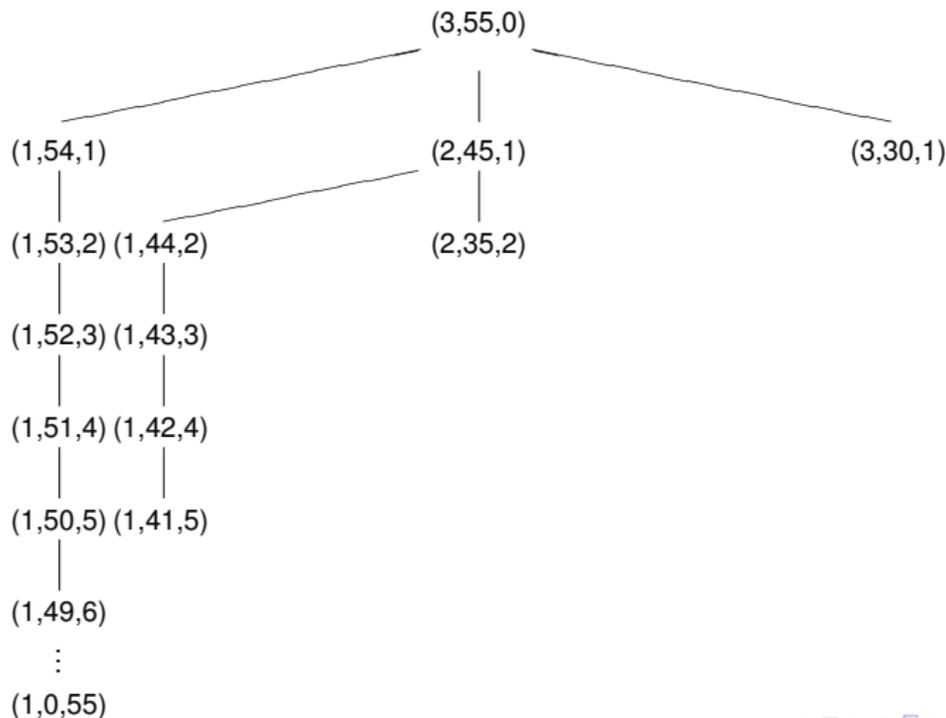
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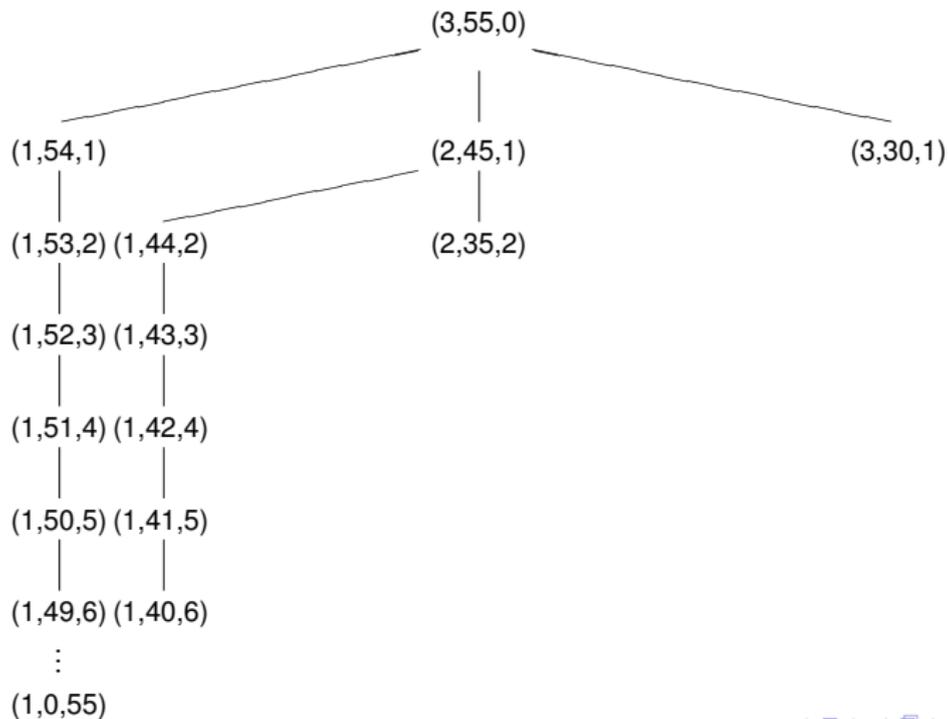
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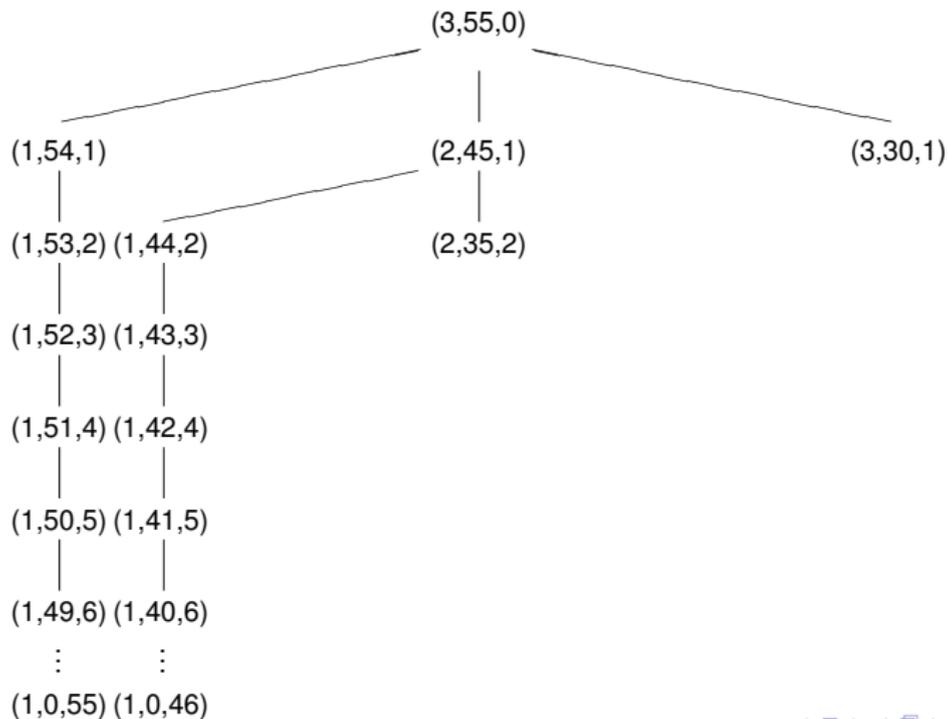
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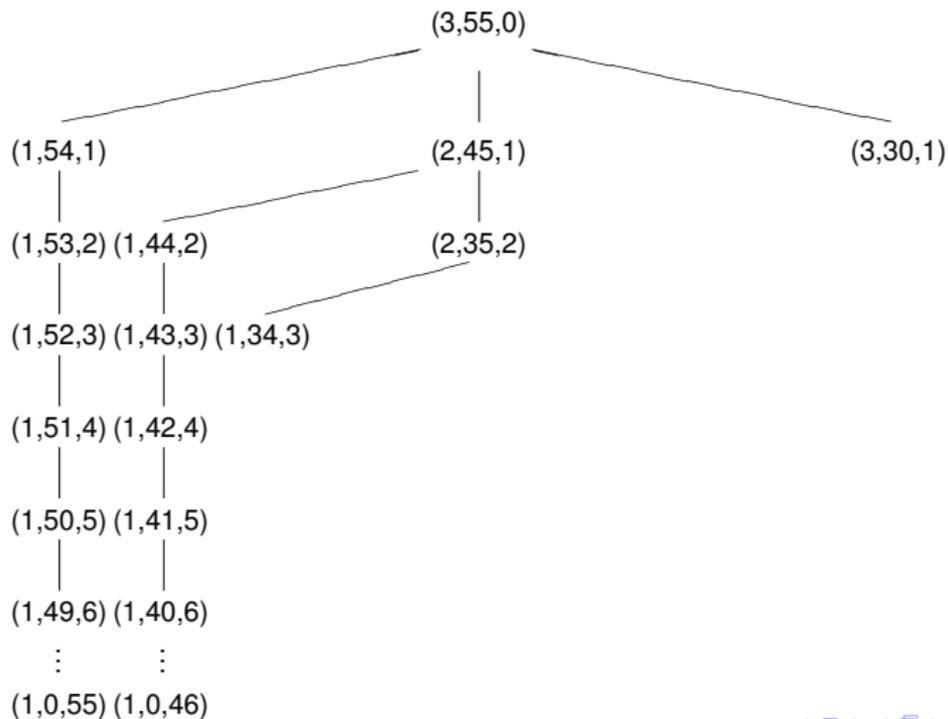
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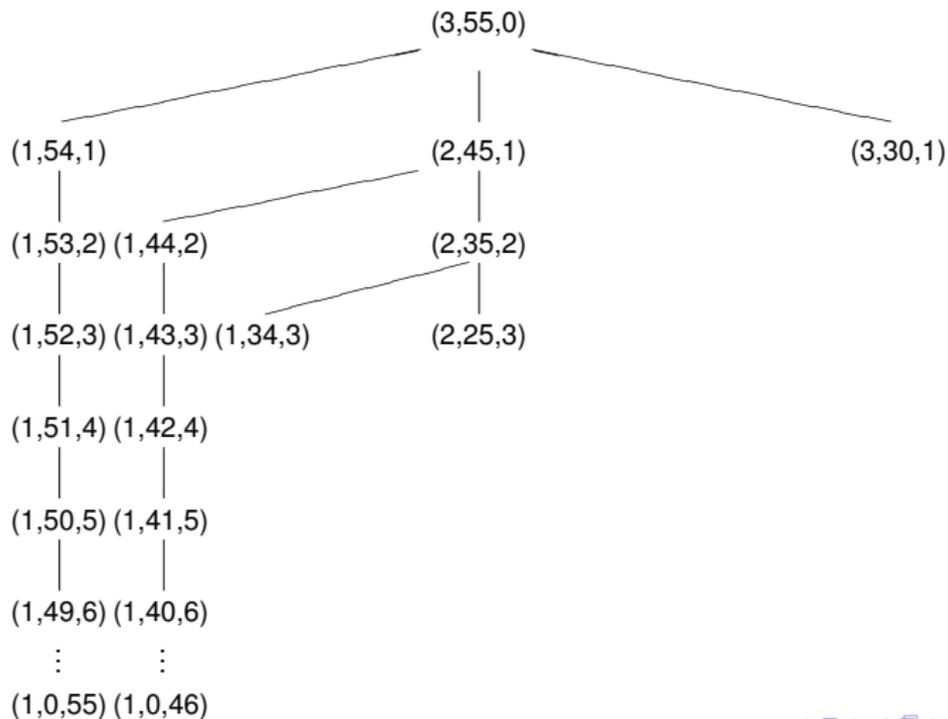
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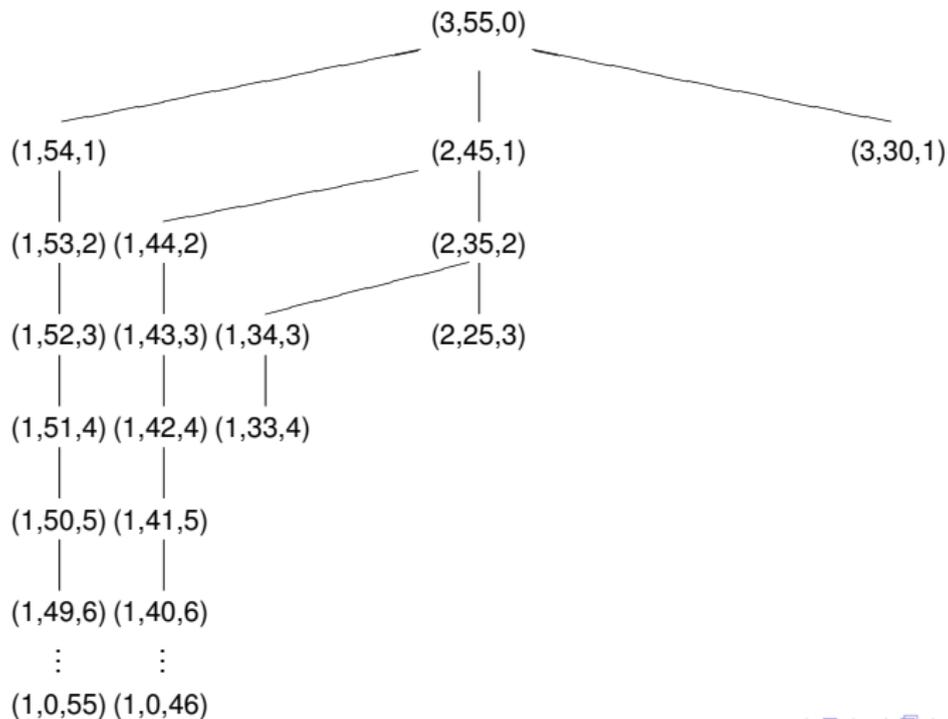
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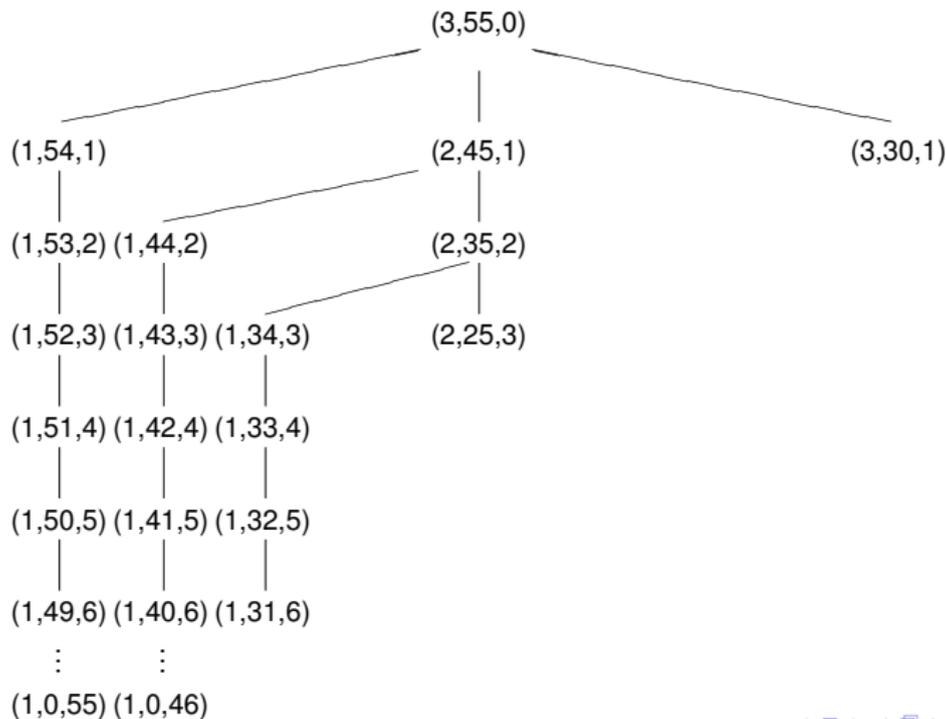
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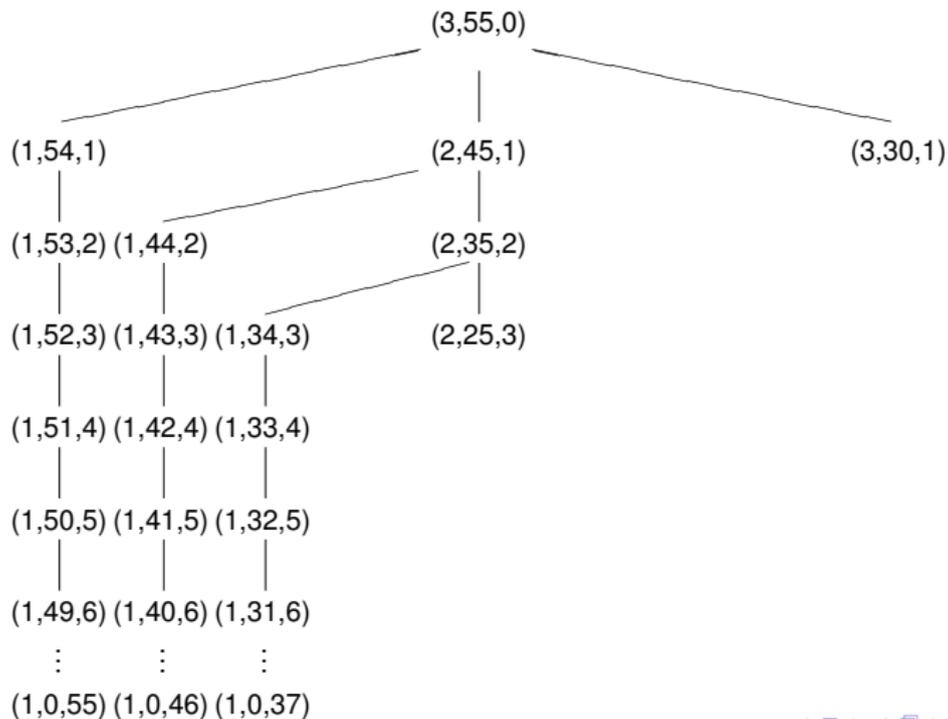
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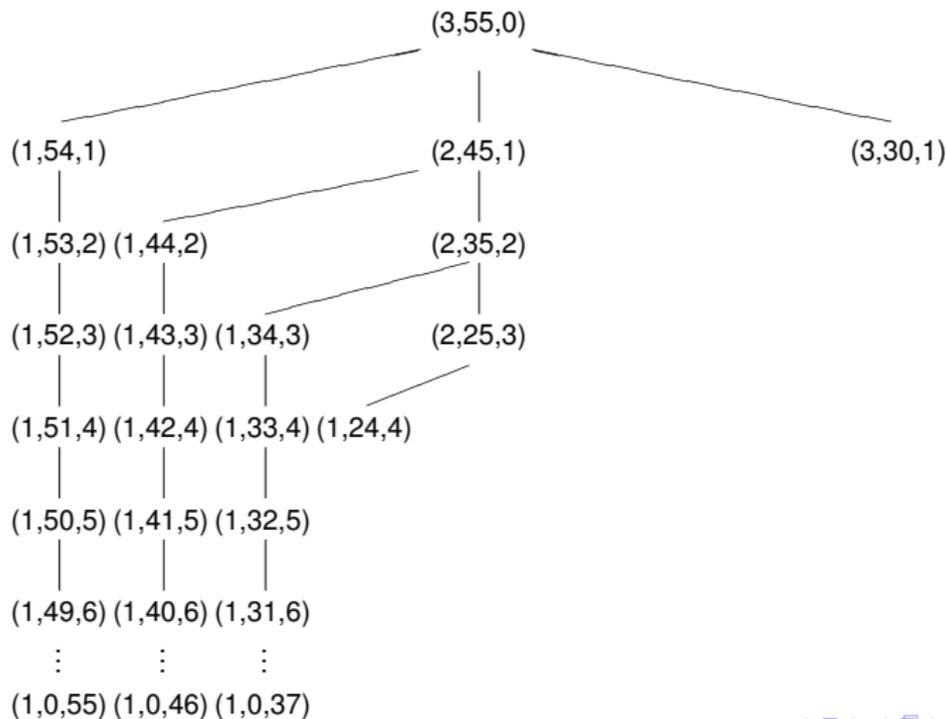
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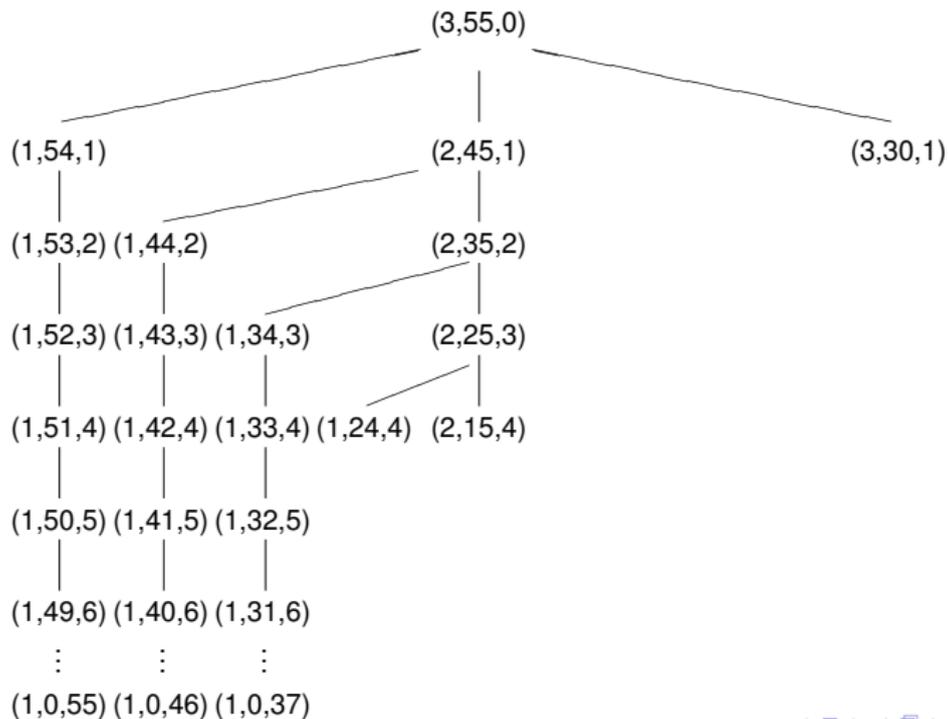
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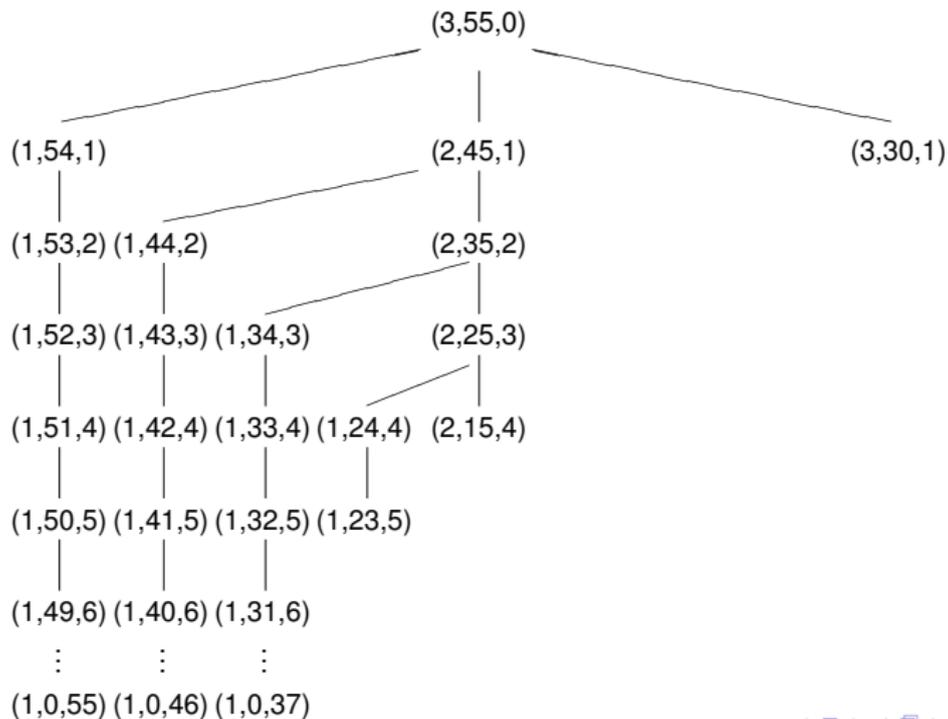
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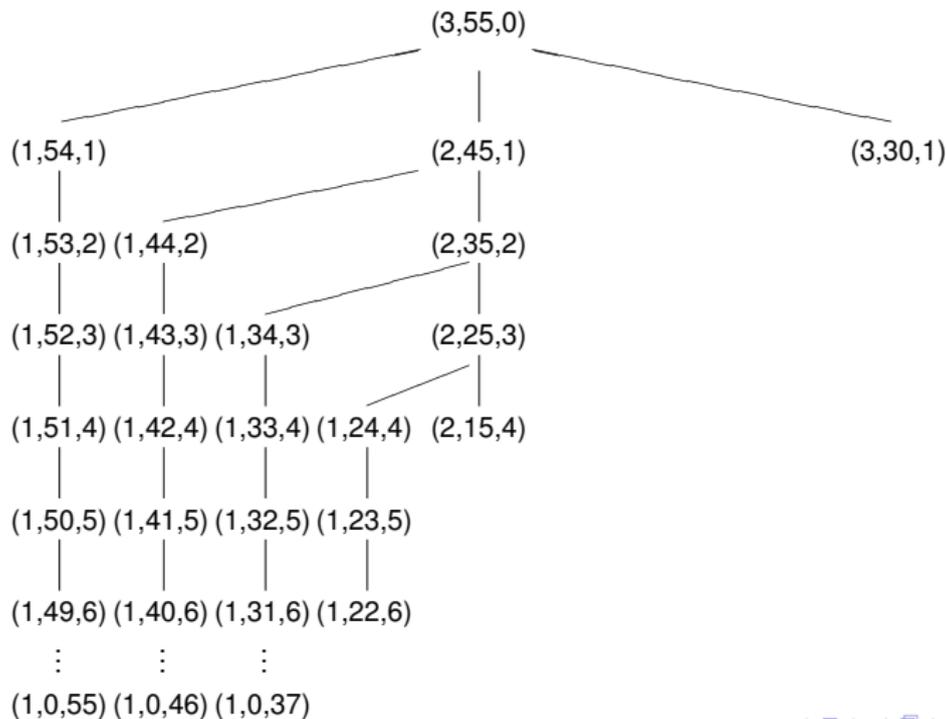
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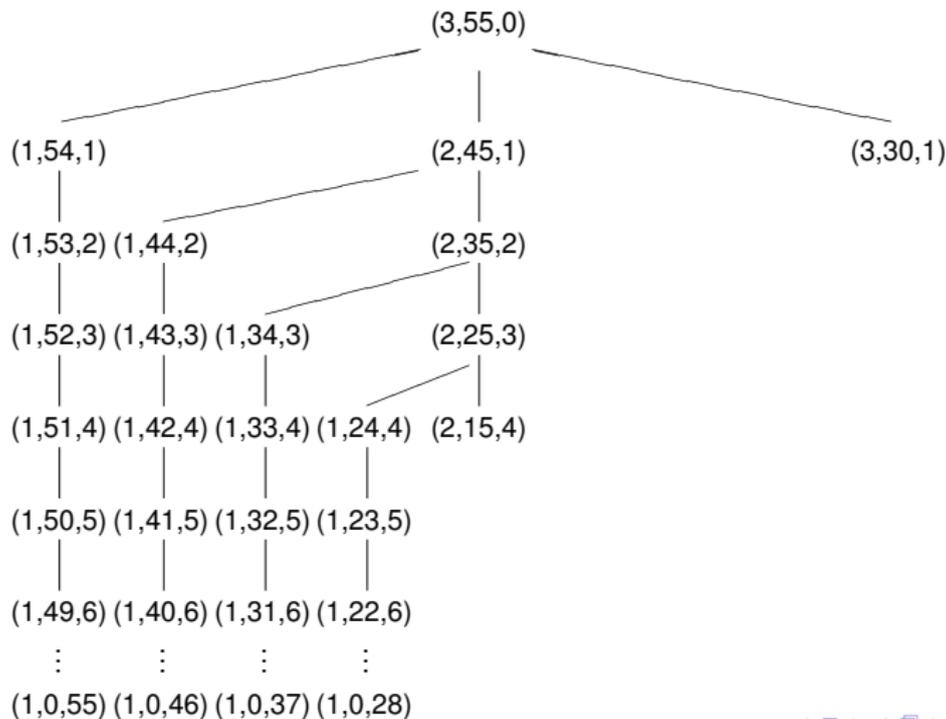
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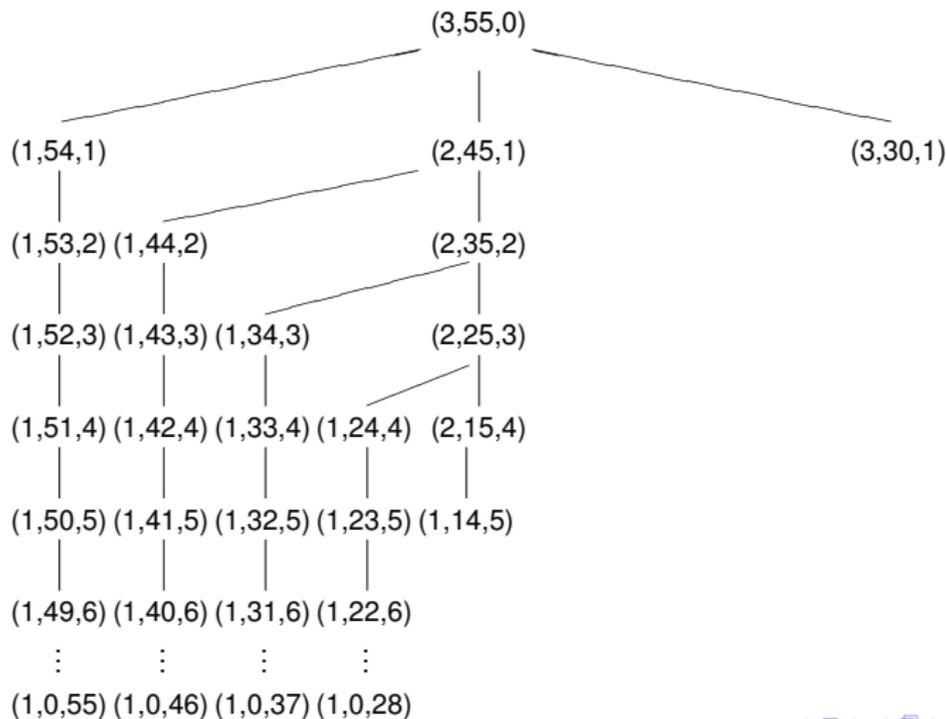
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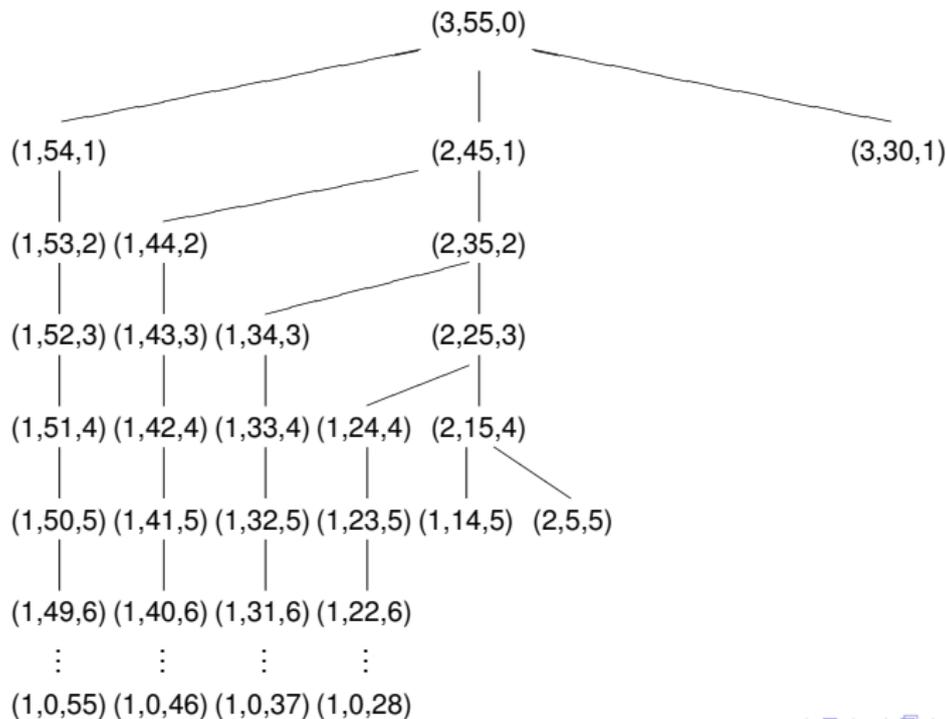
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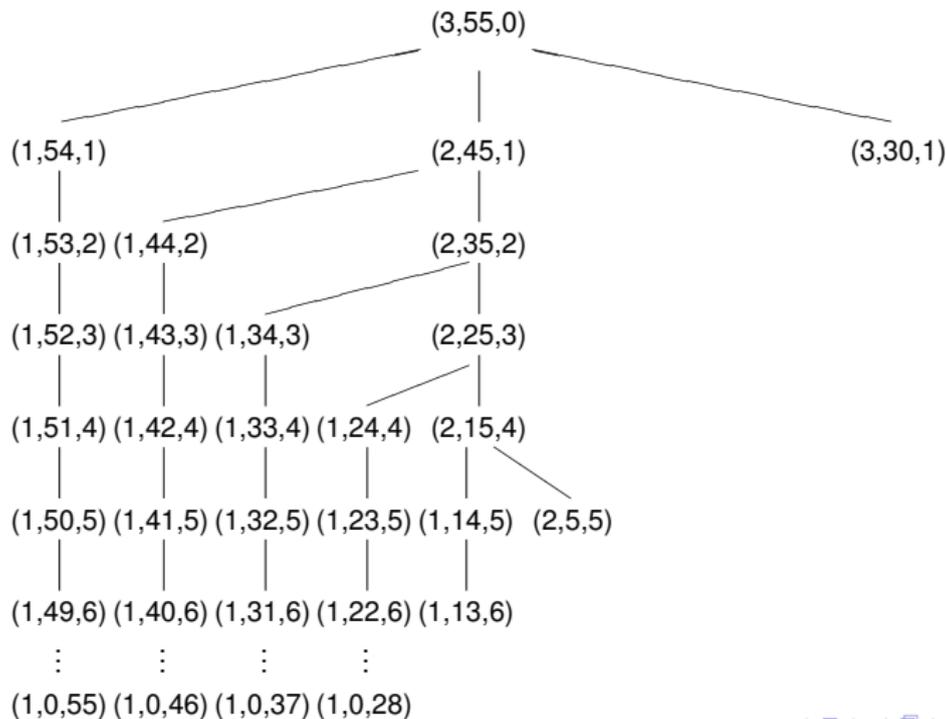
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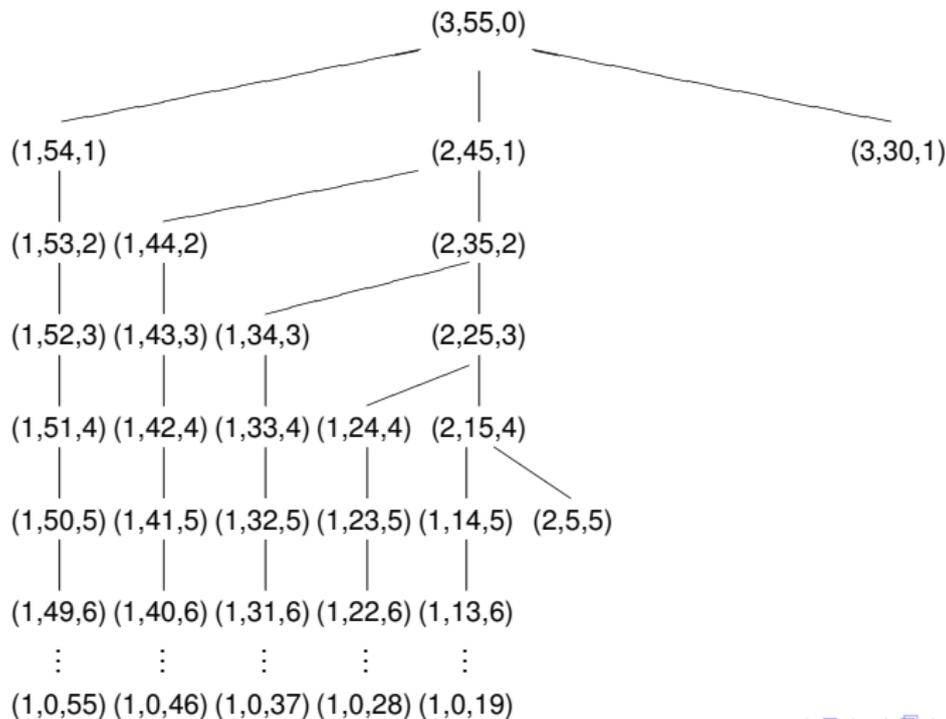
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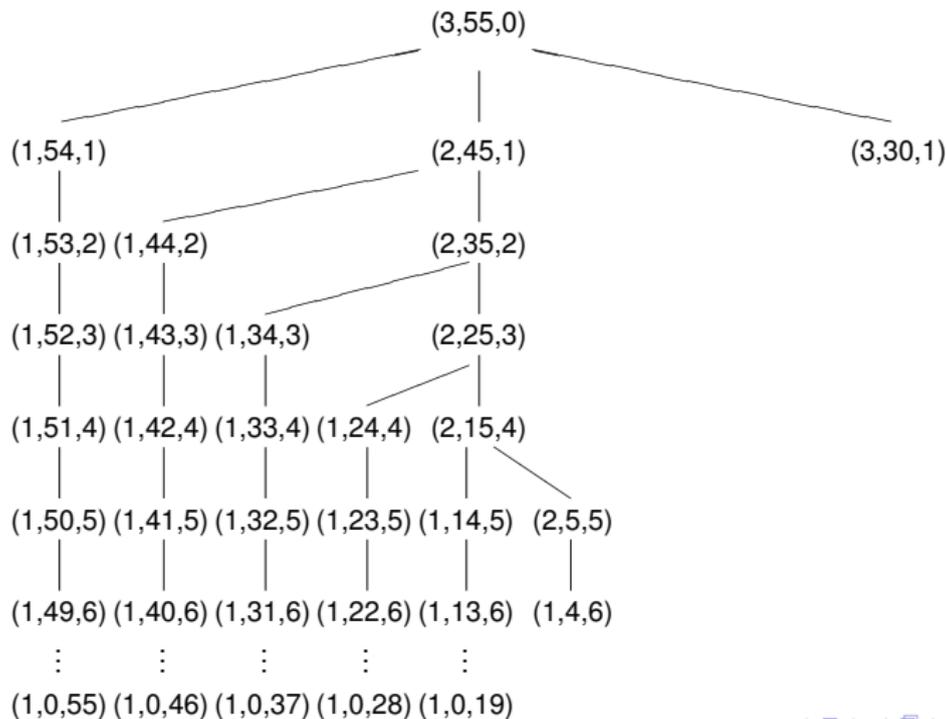
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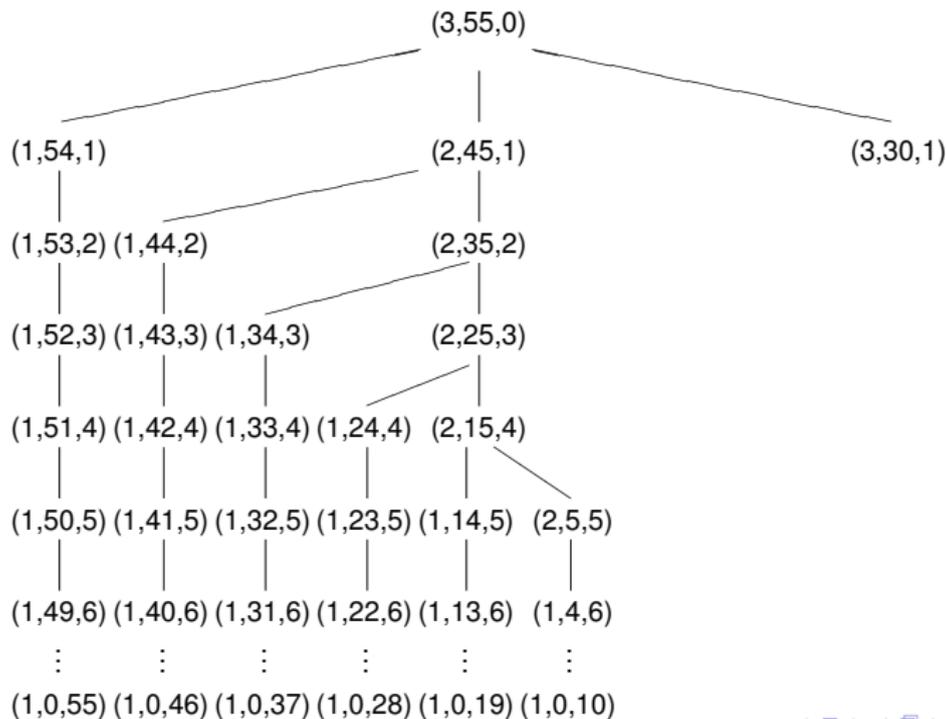
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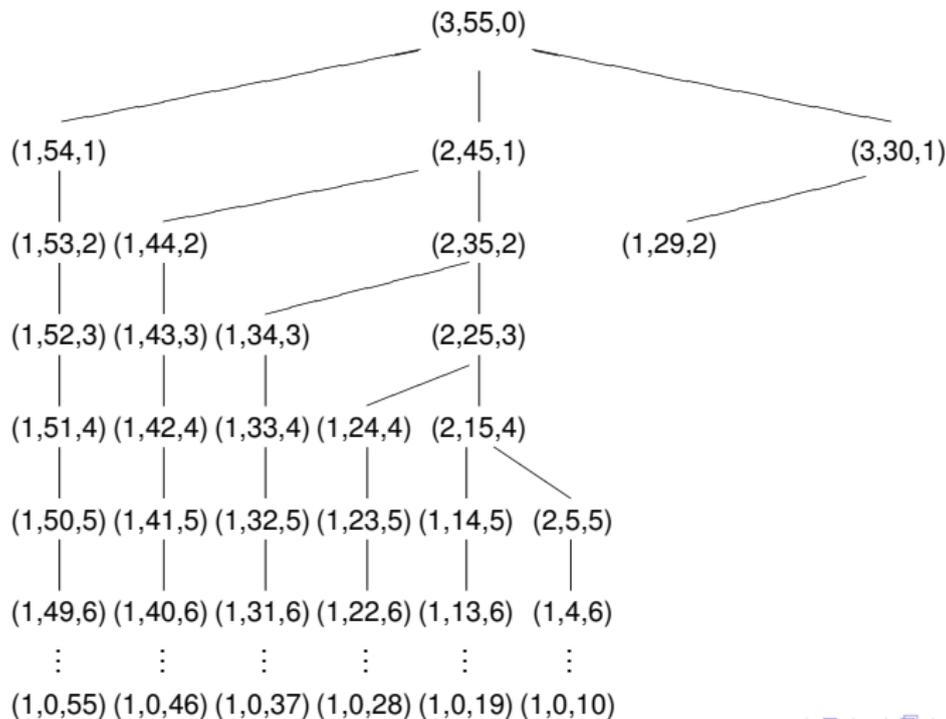
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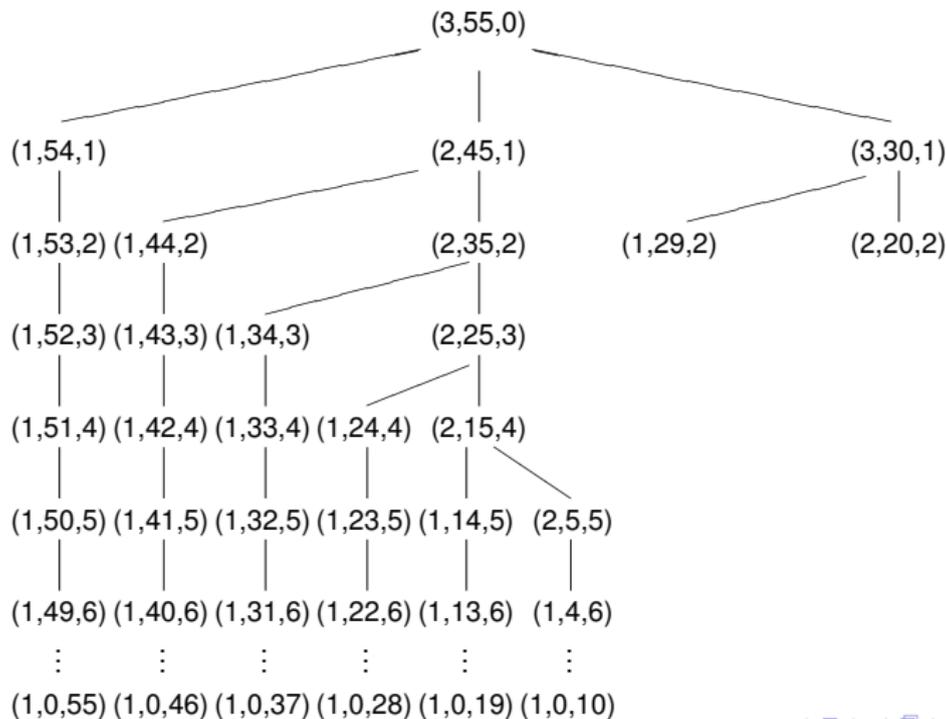
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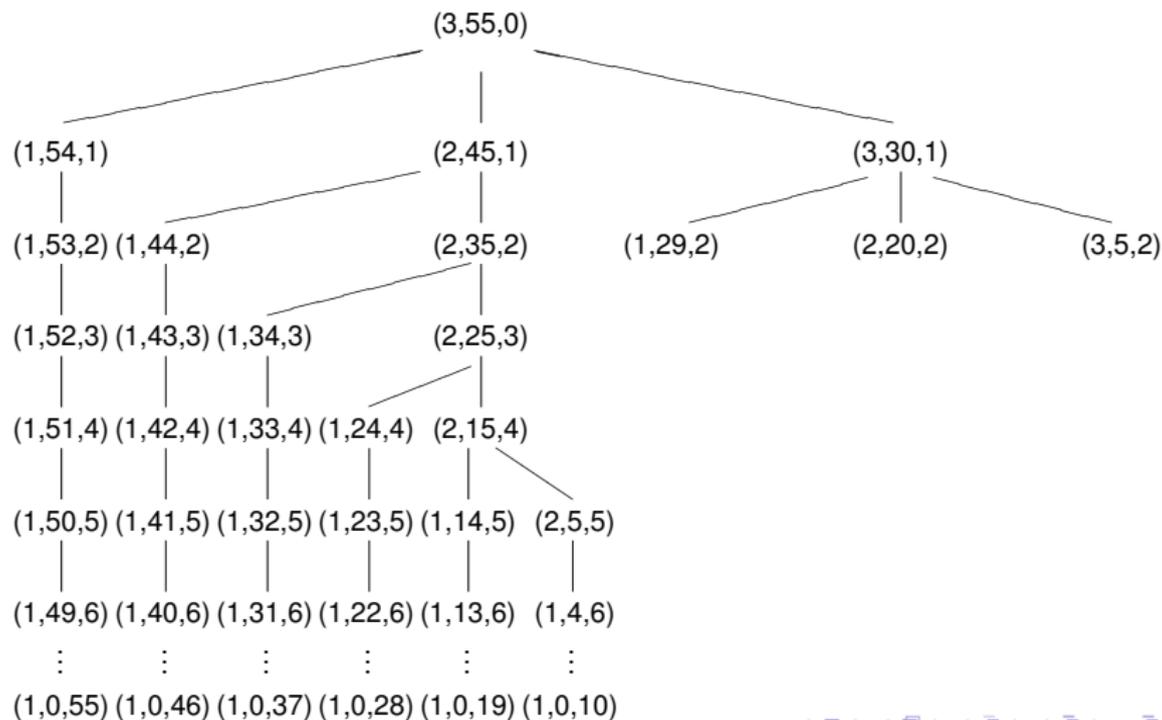
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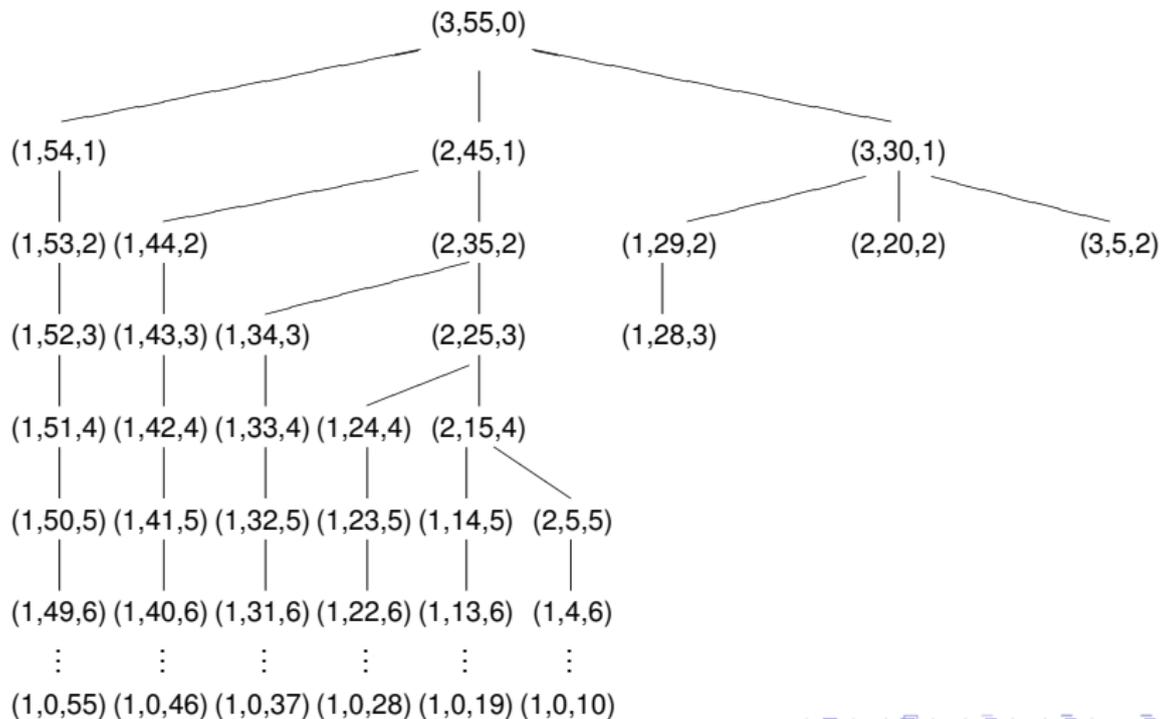
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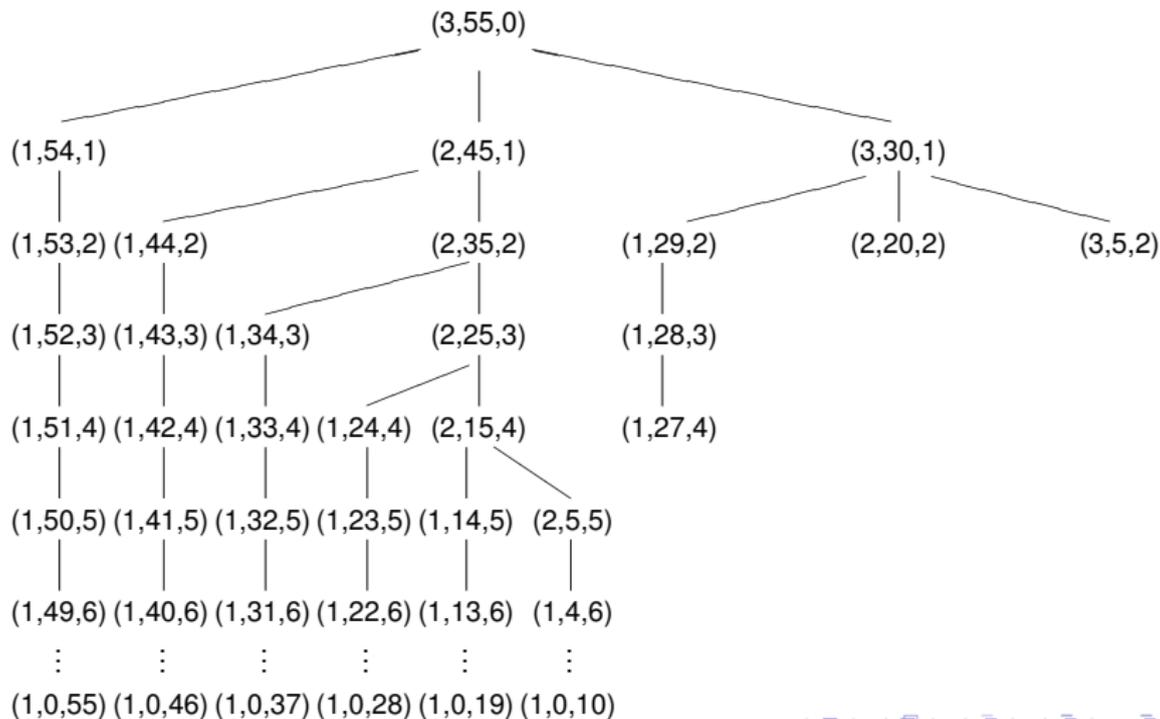
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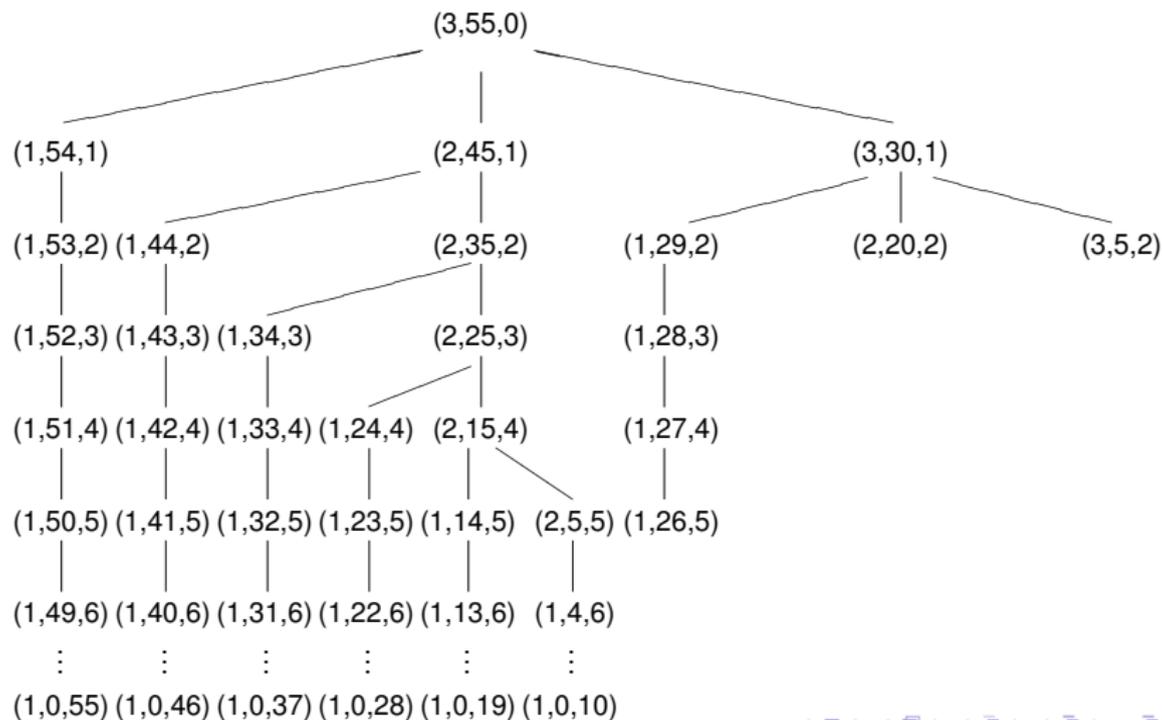
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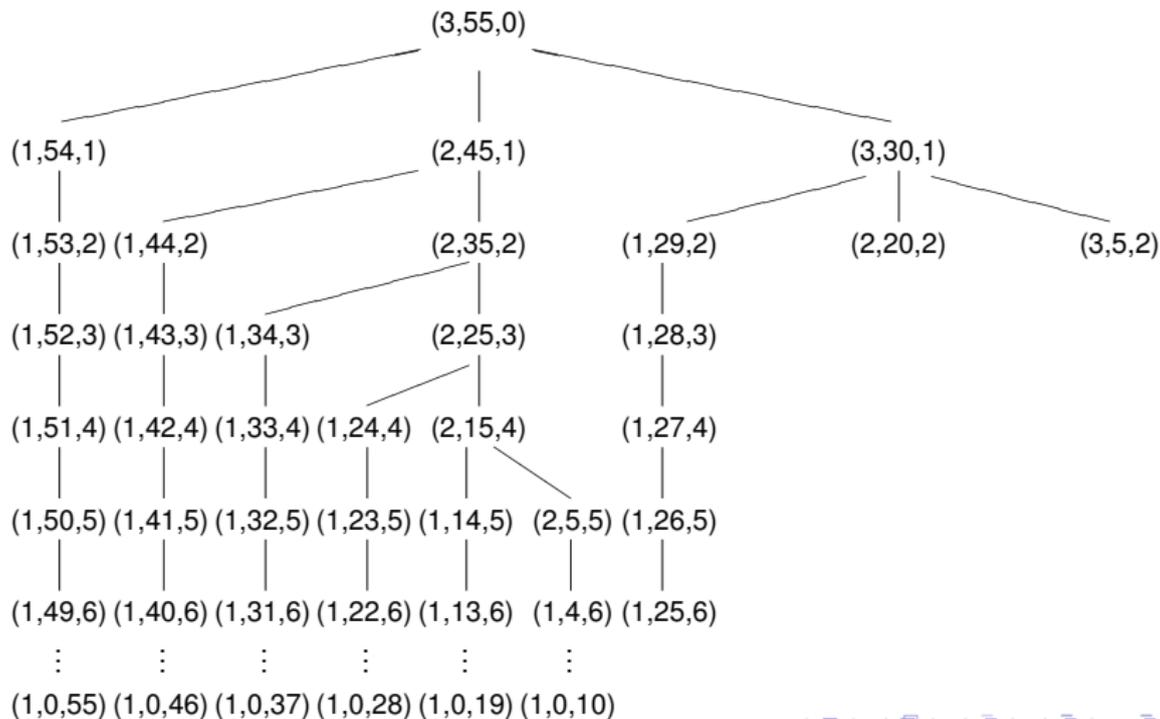
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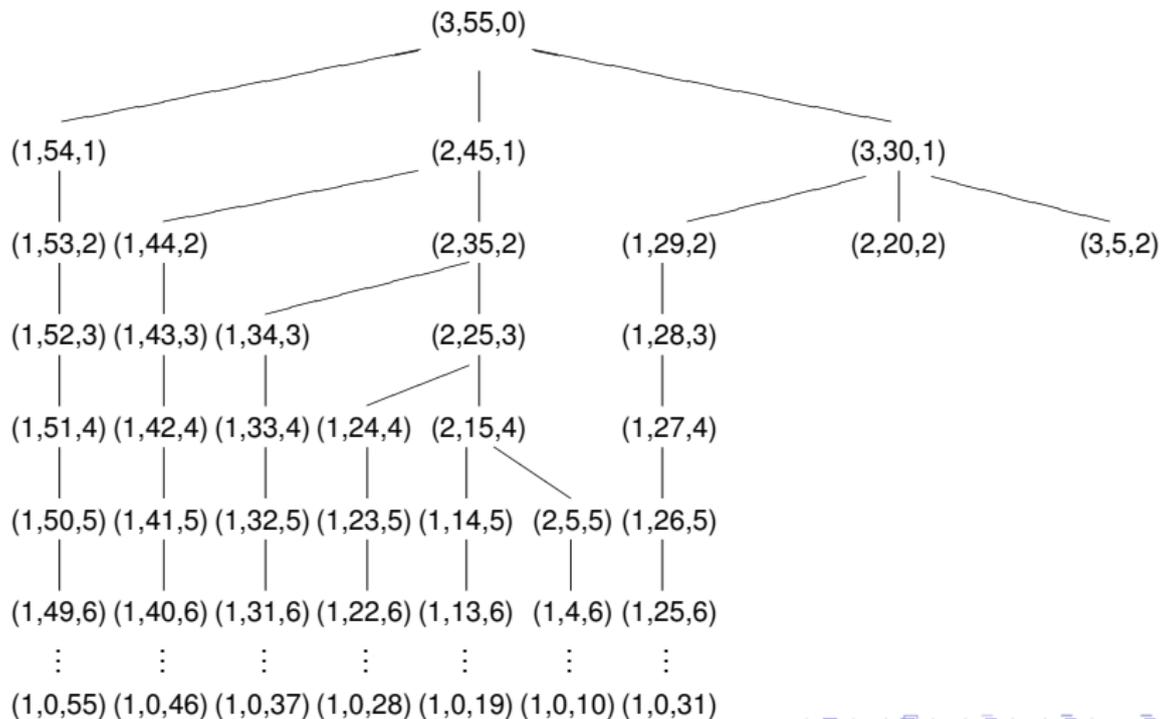
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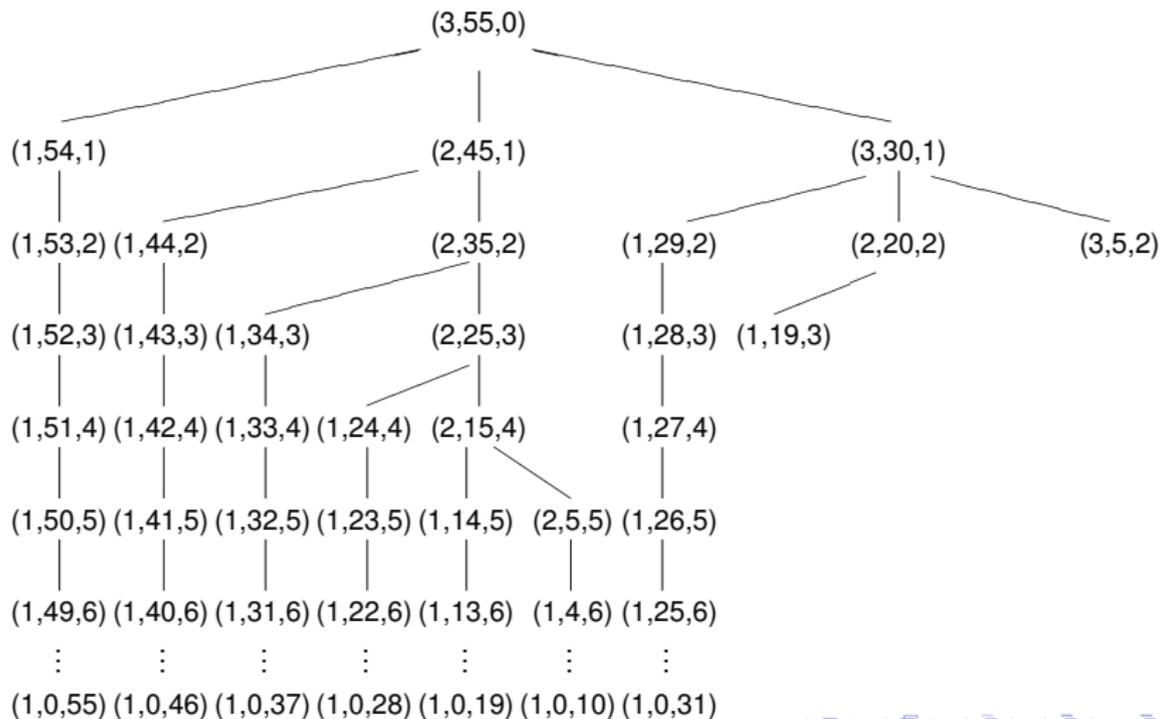
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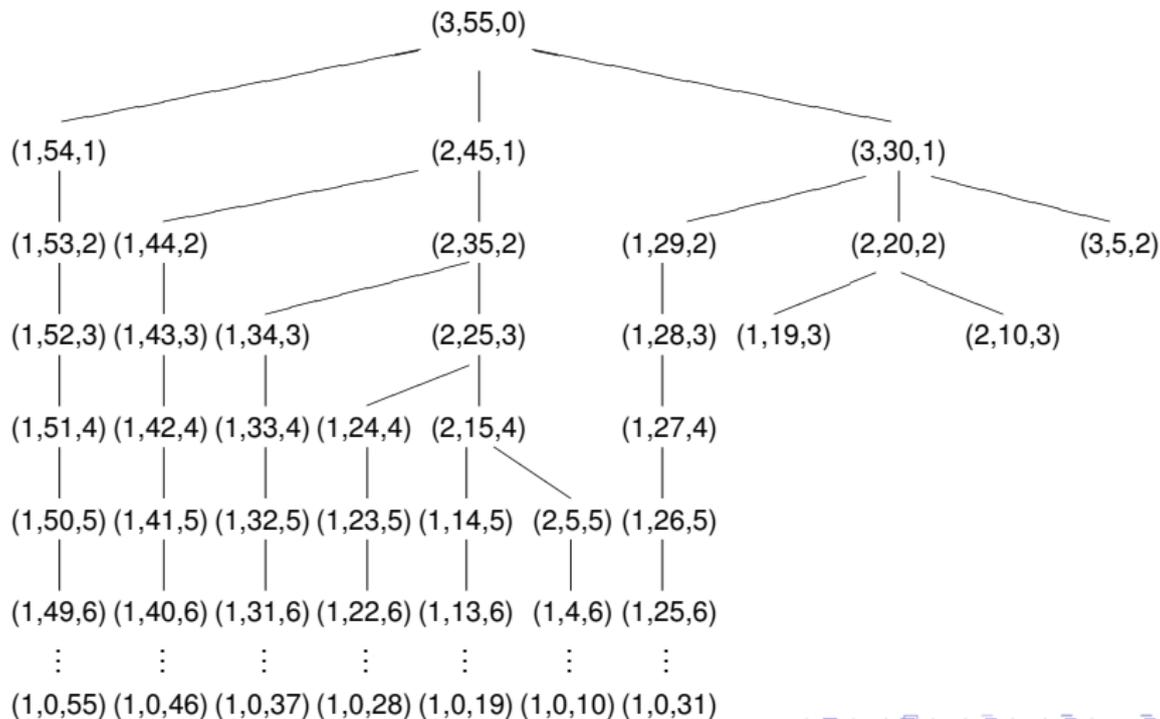
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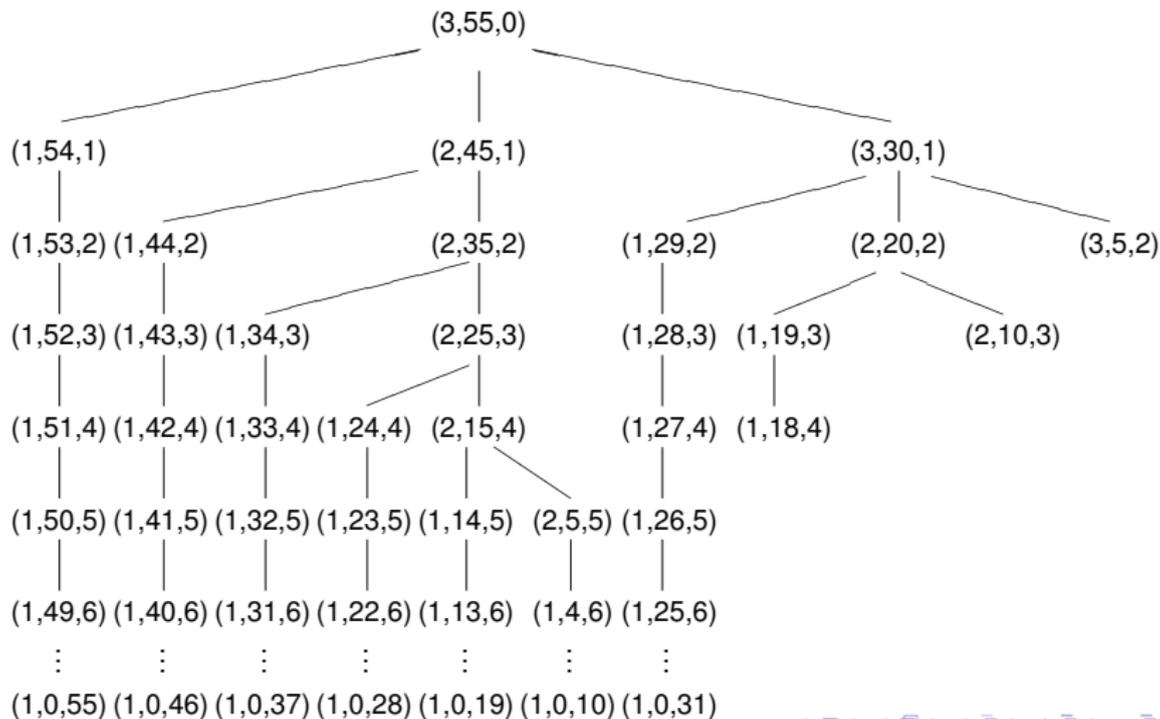
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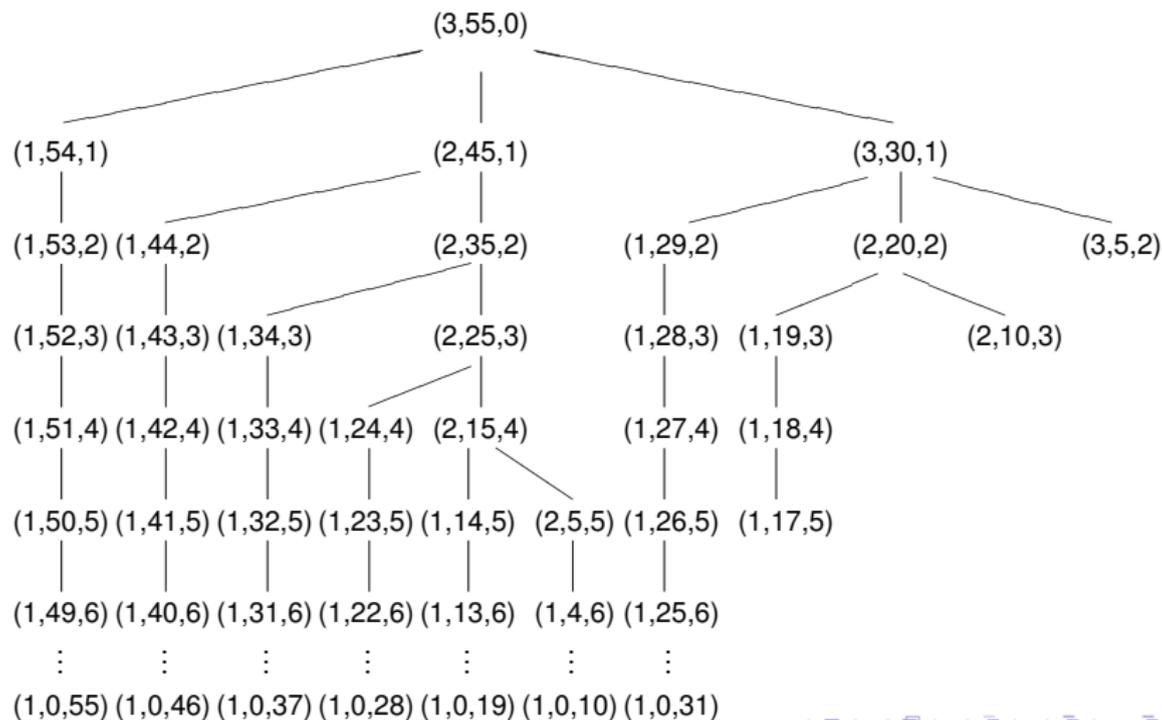
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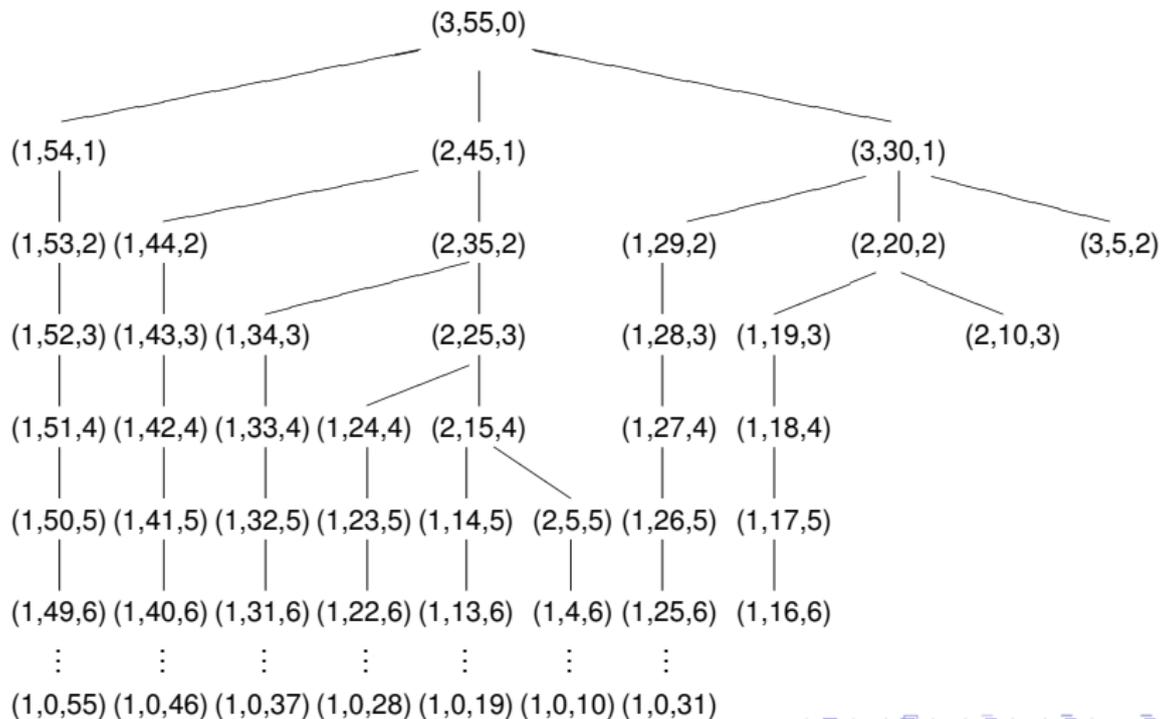
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



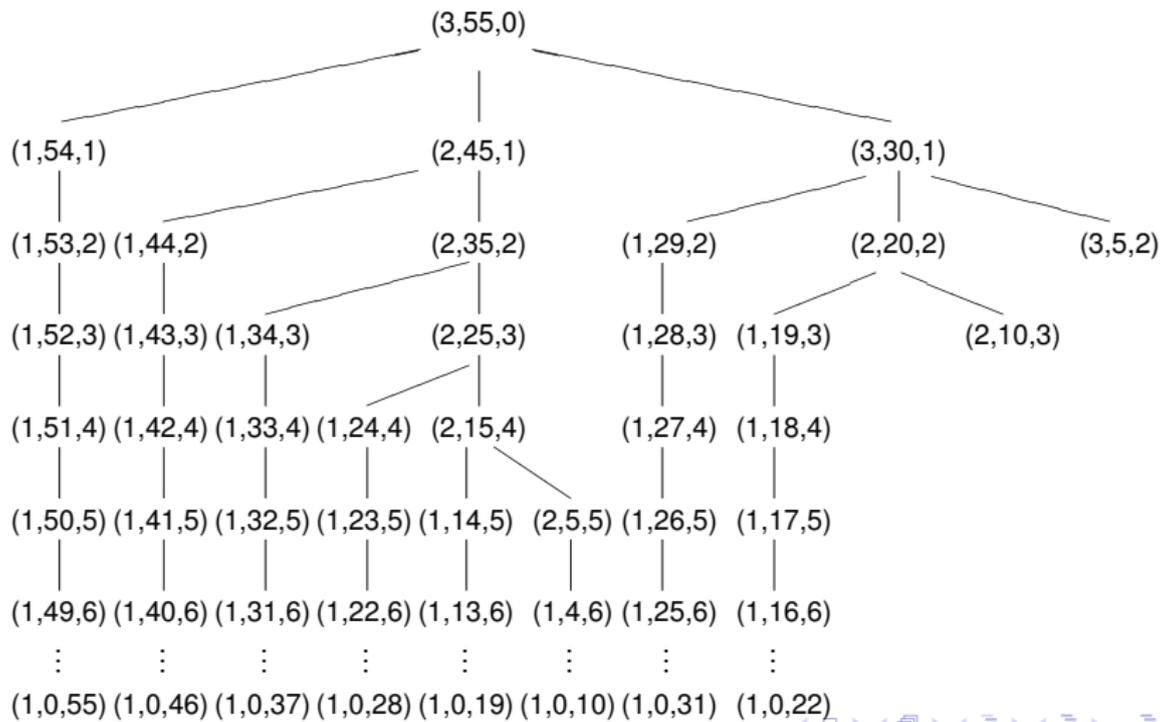
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



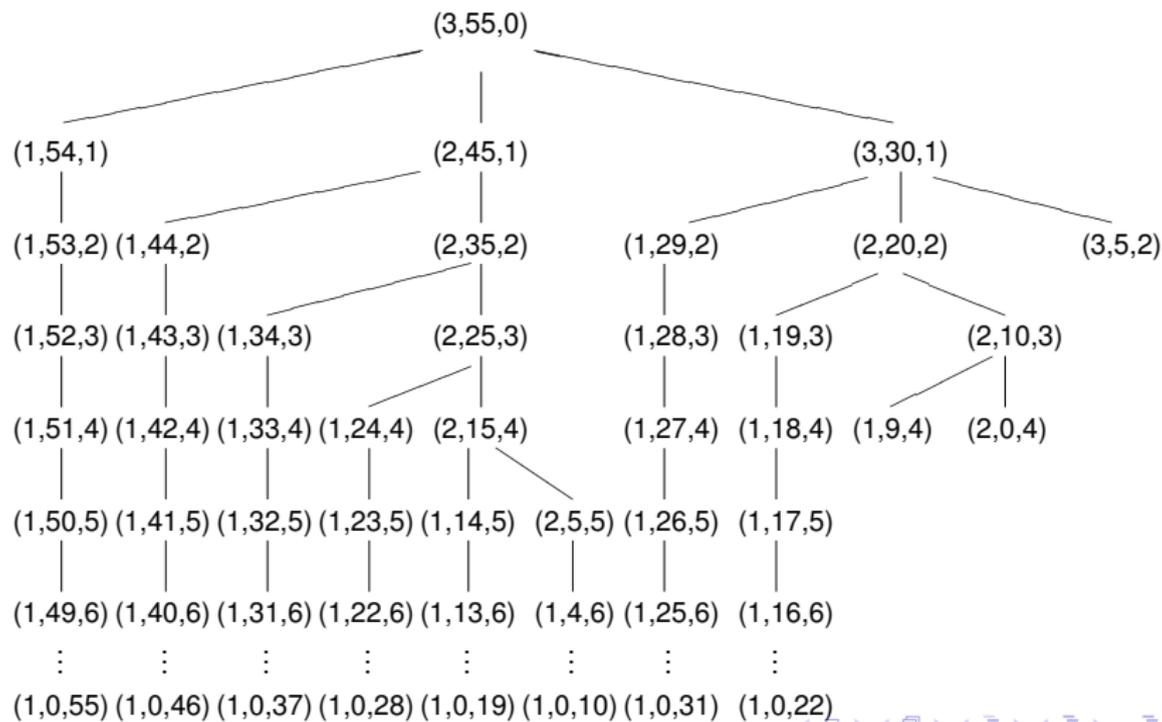
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



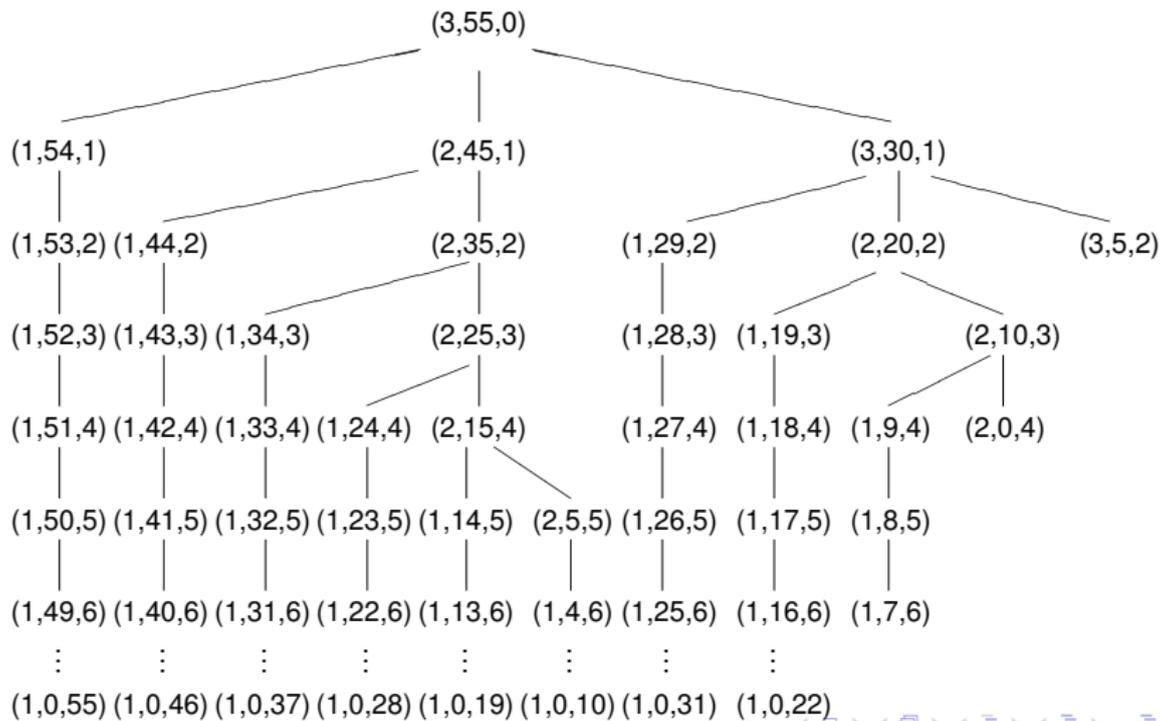
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



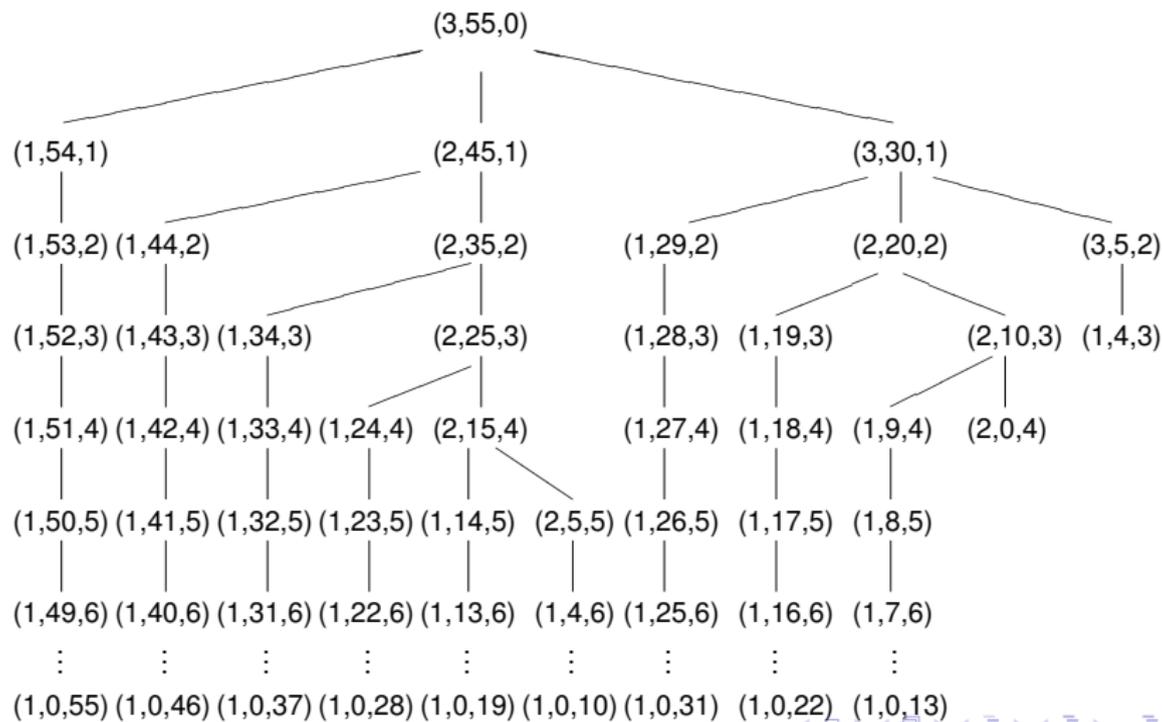
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



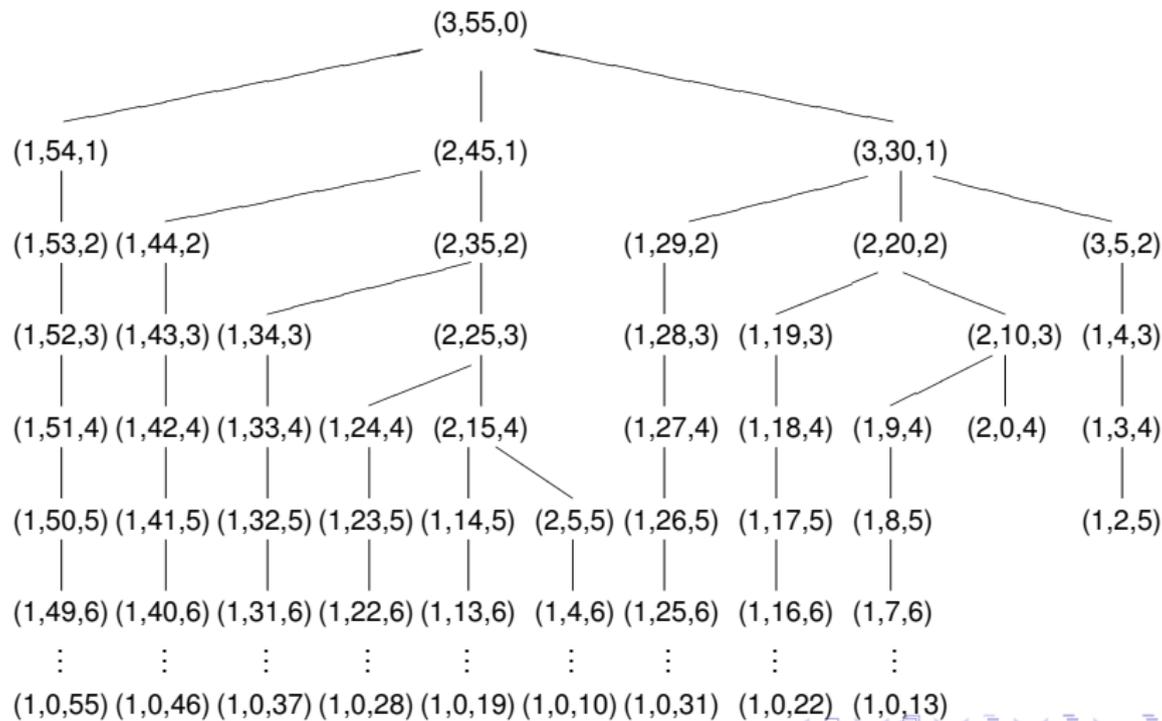
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



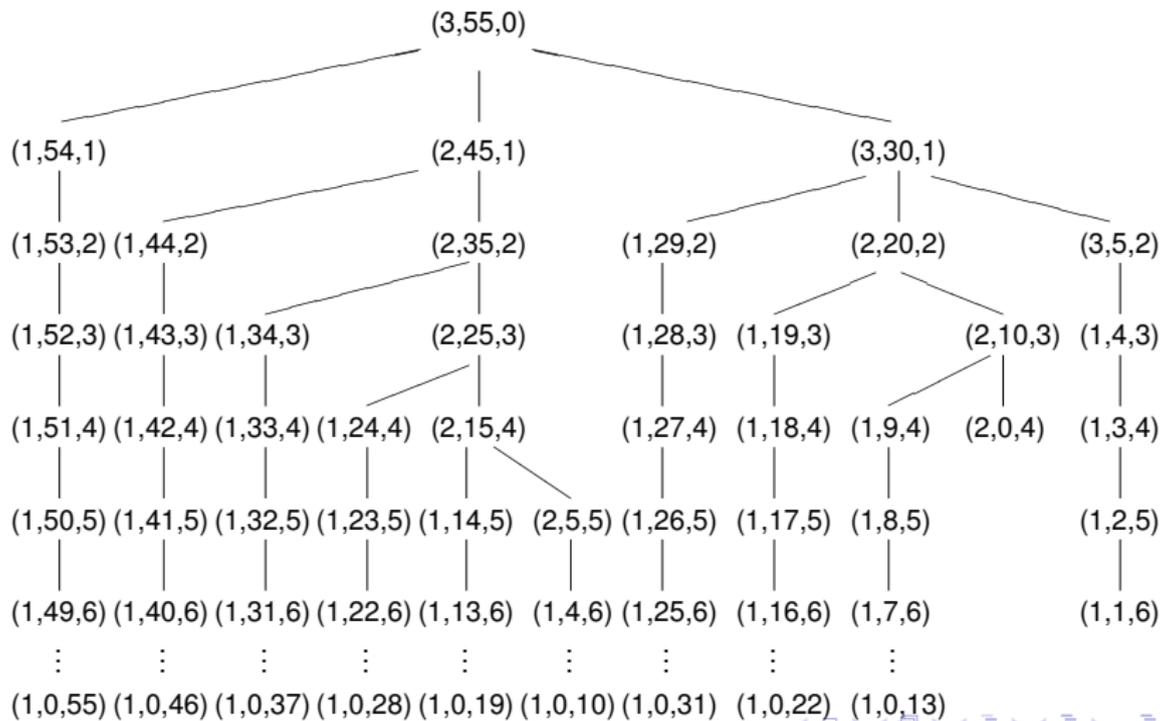
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



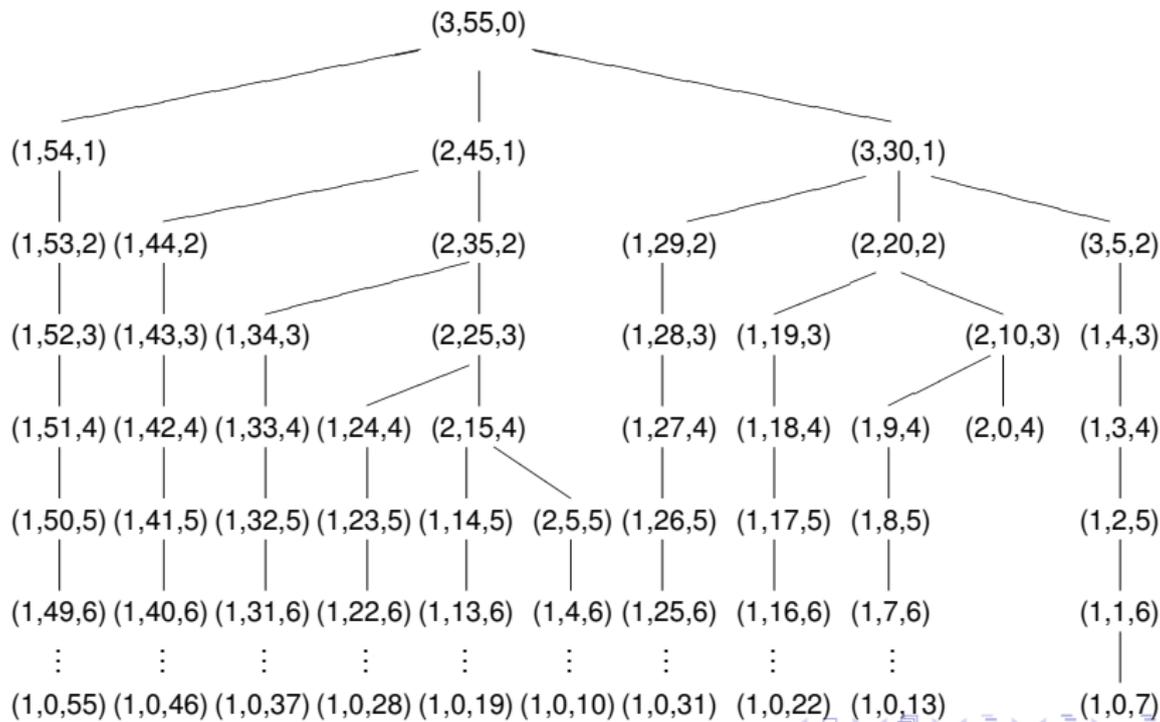
Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



Grafo implícito

Ejemplo $d_1 = 1$, $d_2 = 10$, $d_3 = 25$ y $k = 55$



Grafo implícito

Definición general

- La raíz resulta la misma que en el caso anterior,
- pero el vértice (i, j, x) puede tener 0, 1, o varios hijos:
 - todos los vértices de la forma $(i', j - d_{i'}, 1 + x)$ tal que $1 \leq i' \leq i$ y $d_{i'} \leq j$,
 - son hijos de (i, j, x) .

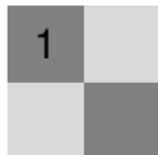
Ocho reinas

- Problema: Encontrar la manera de ubicar 8 reinas en un tablero de 8 filas por 8 columnas de manera tal que ningún par de reinas ocupe la misma fila, la misma columna o la misma diagonal.
- para los que saben ajedrez: de modo de que ninguna reina amenace a otra.
- Es un ejemplo típico de problema que se resuelve usando backtracking.
- Se puede generalizar: ubicar n reinas en un tablero de n filas por n columnas de manera tal que ningún par de reinas ocupe la misma fila, la misma columna o la misma diagonal.
 - 0 reinas es muy fácil de ubicar en un tablero de 0 filas por 0 columnas.
 - 1 reina es muy fácil de ubicar en un tablero de 1 fila por 1 columna.

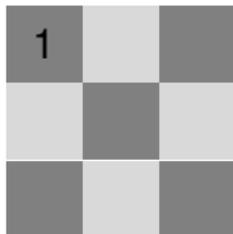
Dos reinas



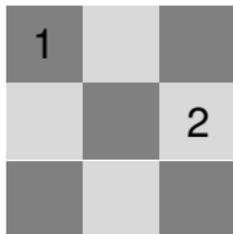
Dos reinas



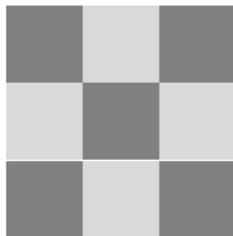
Tres reinas



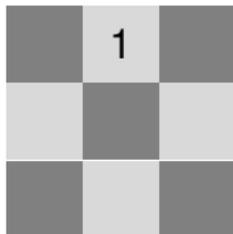
Tres reinas



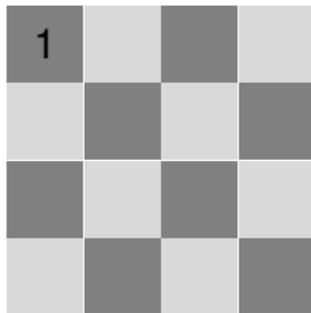
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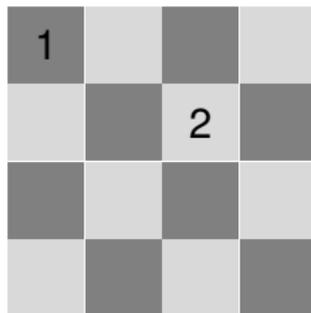
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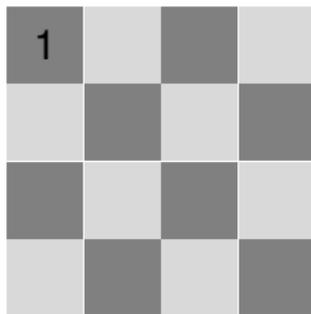
Cuatro reinas



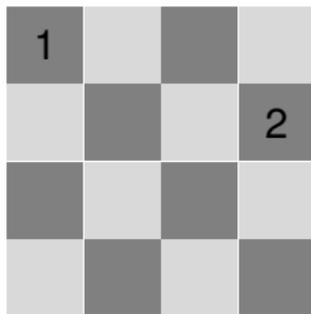
Cuatro reinas



Cuatro reinas



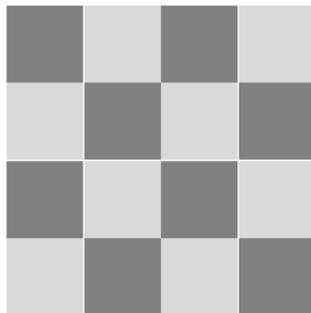
Cuatro reinas



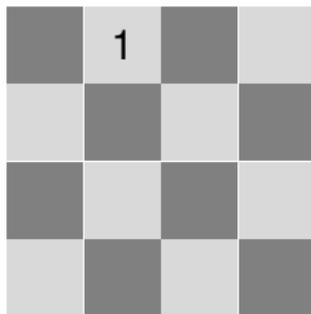
Cuatro reinas

1			
			2
	3		

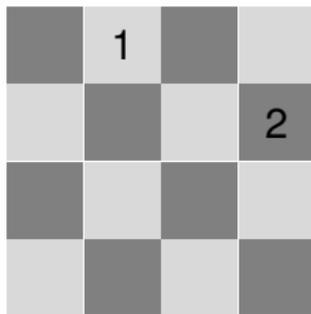
Cuatro reinas



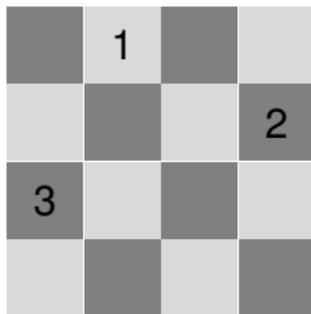
Cuatro reinas



Cuatro reinas



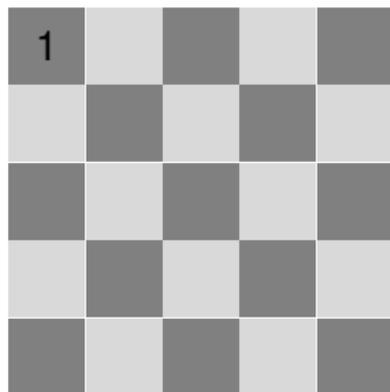
Cuatro reinas



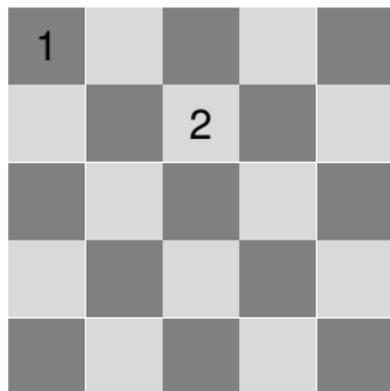
Cuatro reinas

	1		
			2
3			
		4	

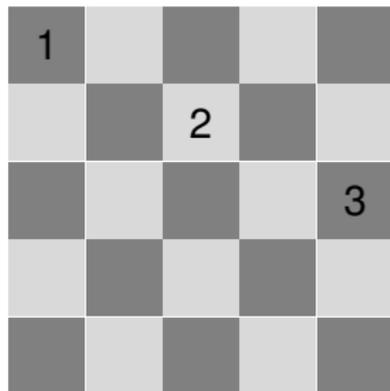
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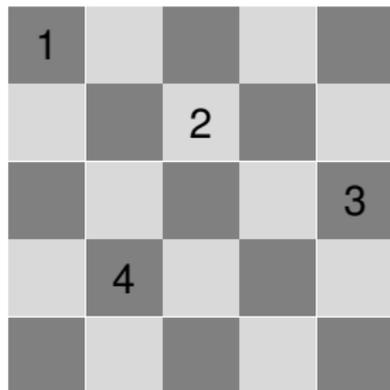
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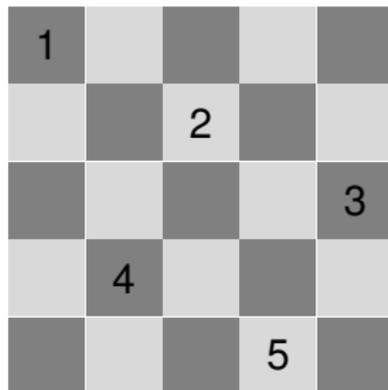
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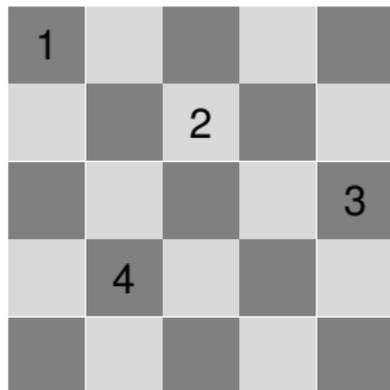
Cinco reinas



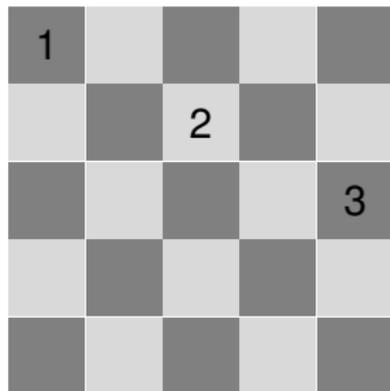
Cinco reinas



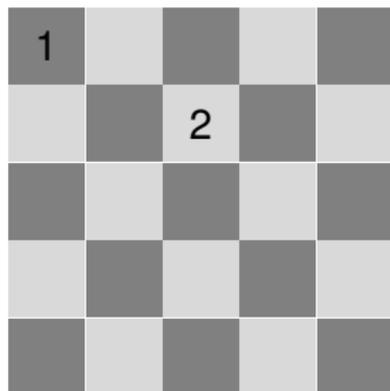
Cinco reinas



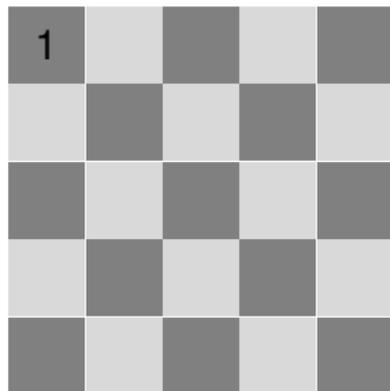
Cinco reinas



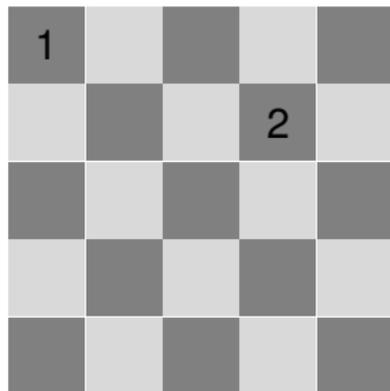
Cinco reinas



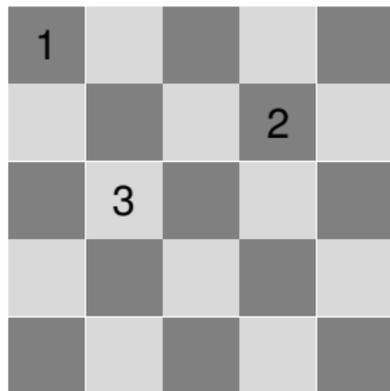
Cinco reinas



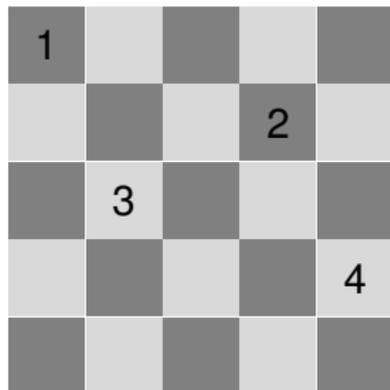
Cinco reinas



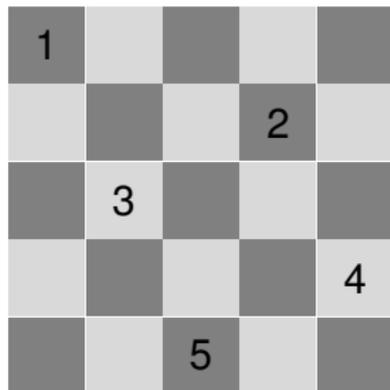
Cinco reinas



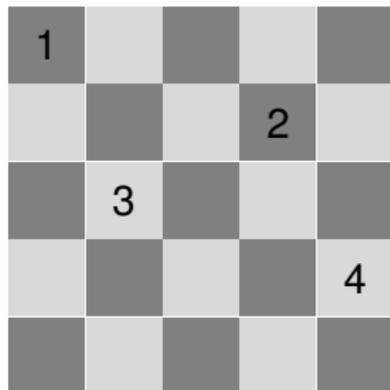
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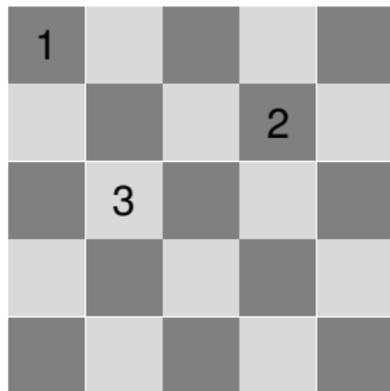
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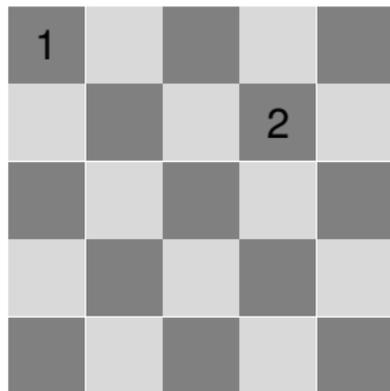
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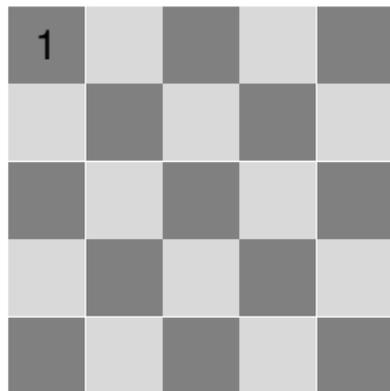
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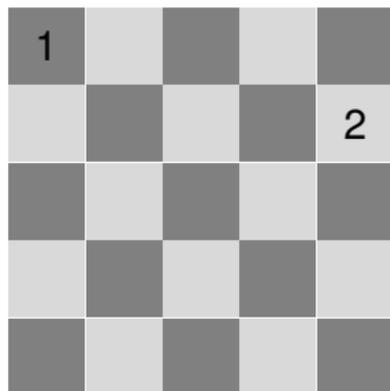
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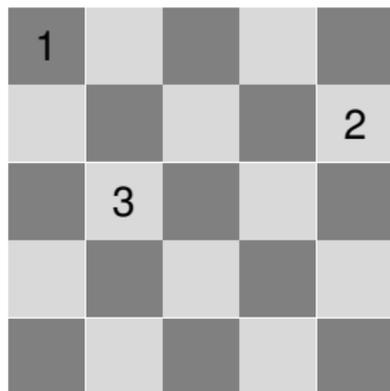
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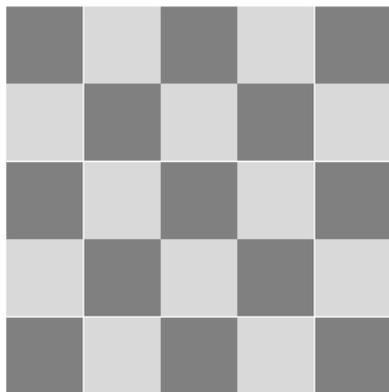
Cinco reinas



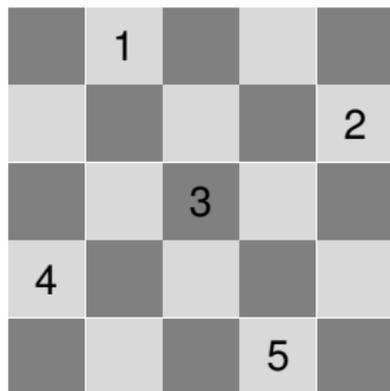
Cinco reinas



Cinco reinas



Cinco reinas

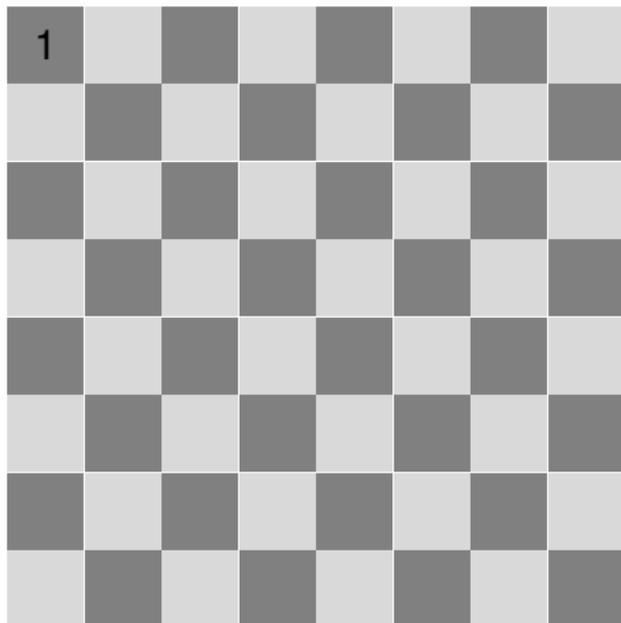


Resumiendo

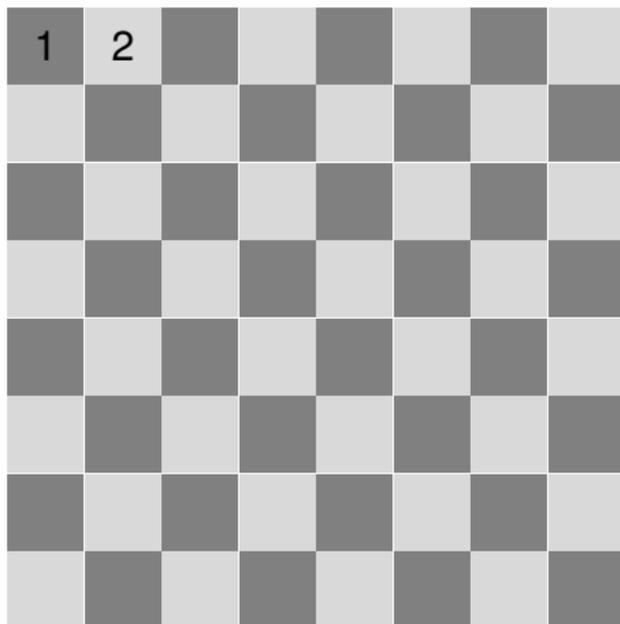
n reinas:

- $n = 0$ tiene una solución
- $n = 1$ tiene una solución
- $n = 2$ no tiene solución
- $n = 3$ no tiene solución
- $n = 4$ tiene solución
- $n = 5$ varias soluciones
- $n \geq 4$ siempre tiene solución

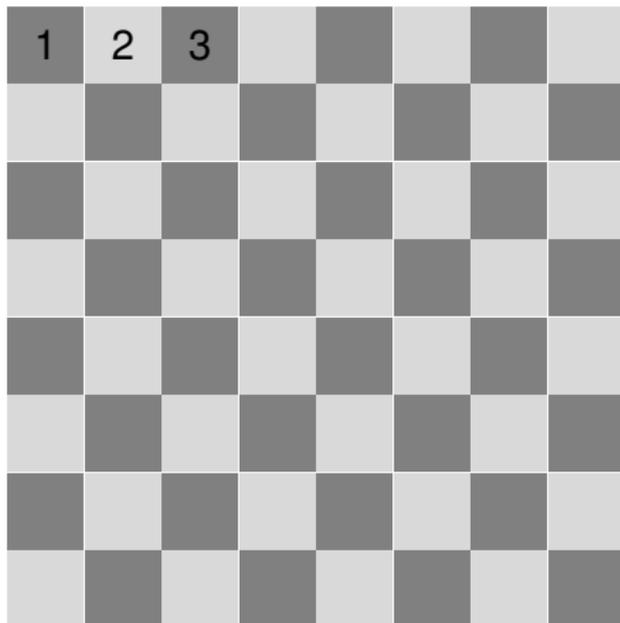
Ocho reinas, peor algoritmo posible



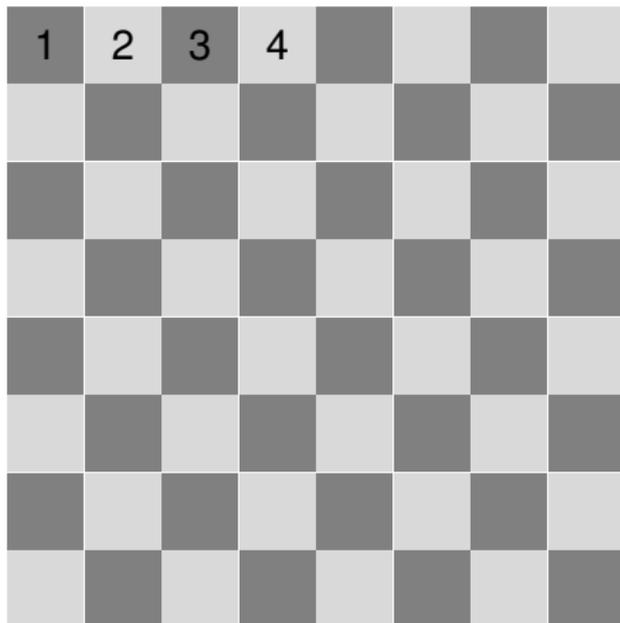
Ocho reinas, peor algoritmo posible



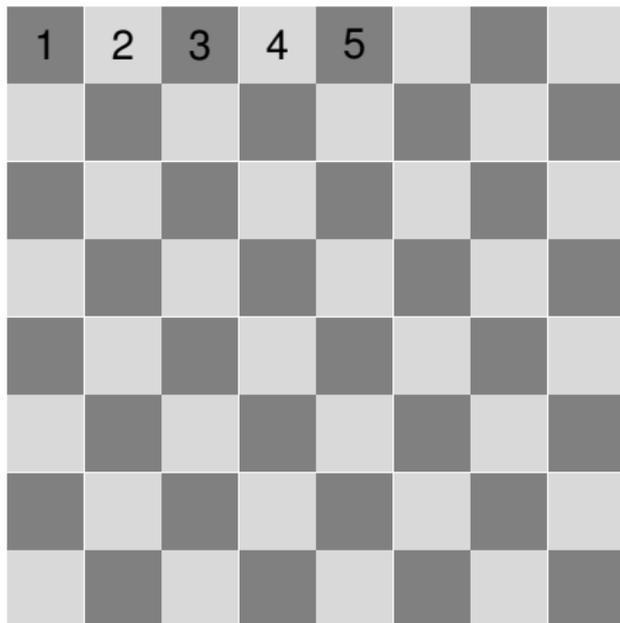
Ocho reinas, peor algoritmo posible



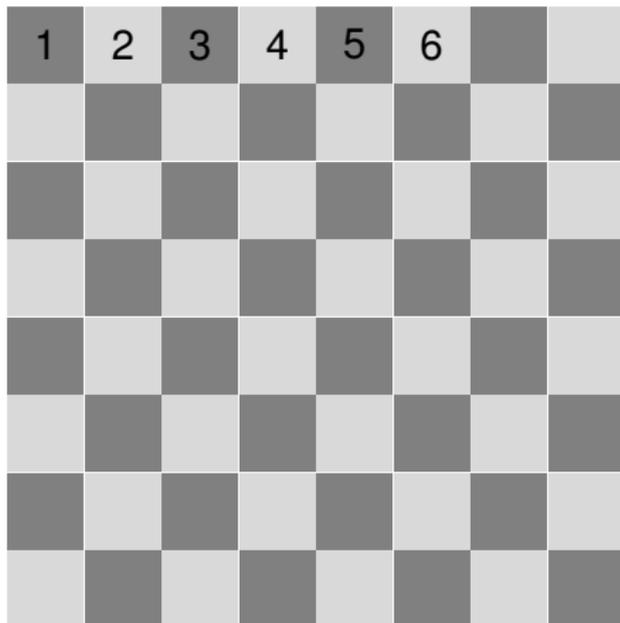
Ocho reinas, peor algoritmo posible



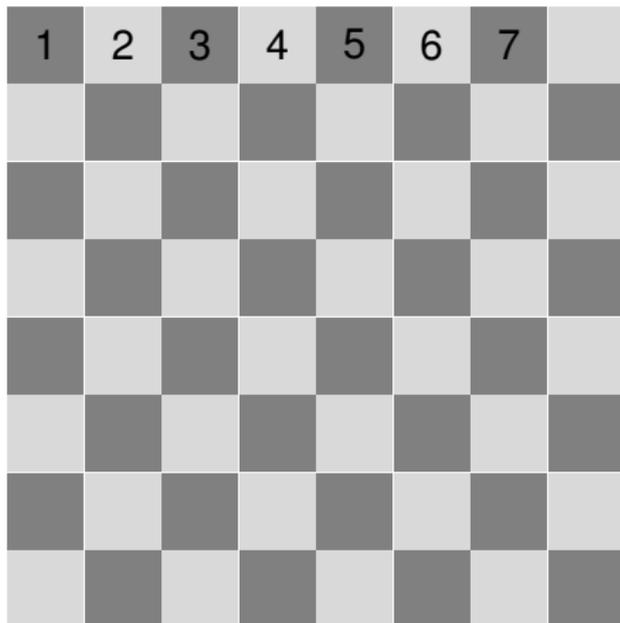
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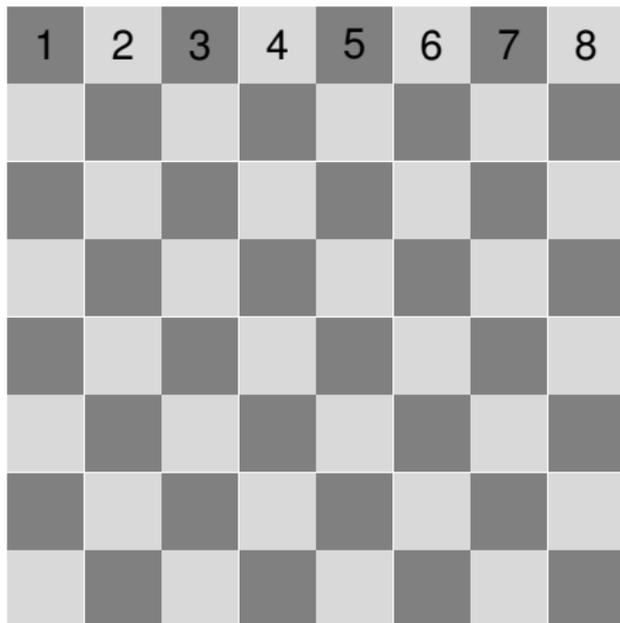
Ocho reinas, peor algoritmo posible



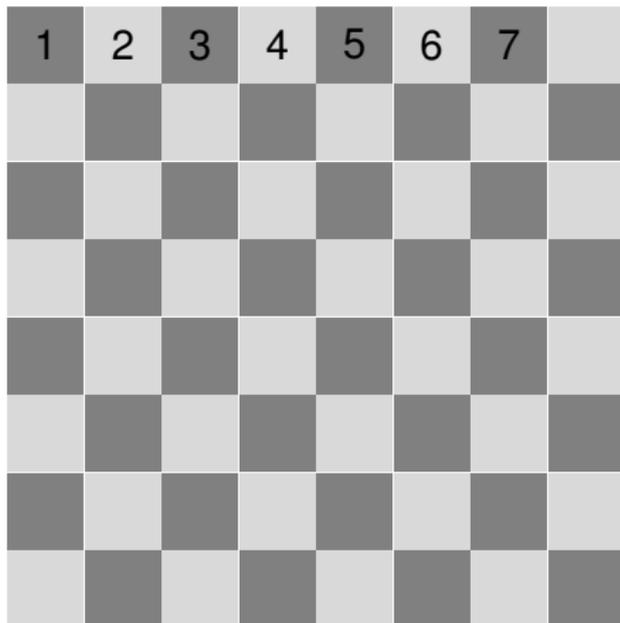
Ocho reinas, peor algoritmo posible



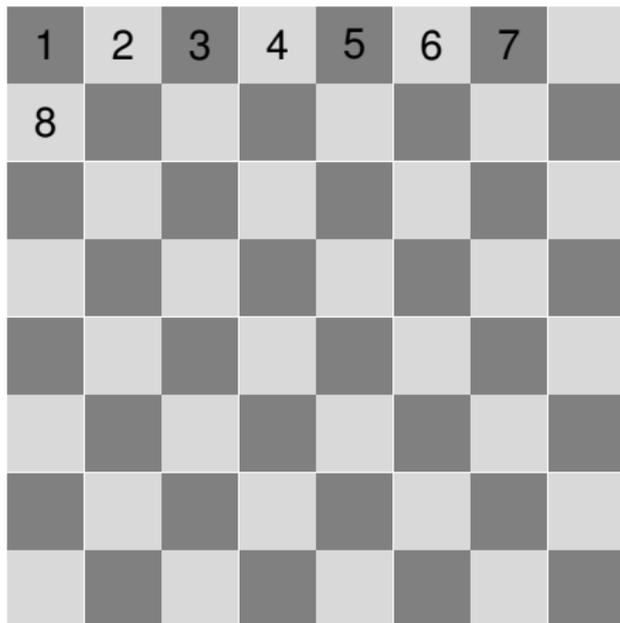
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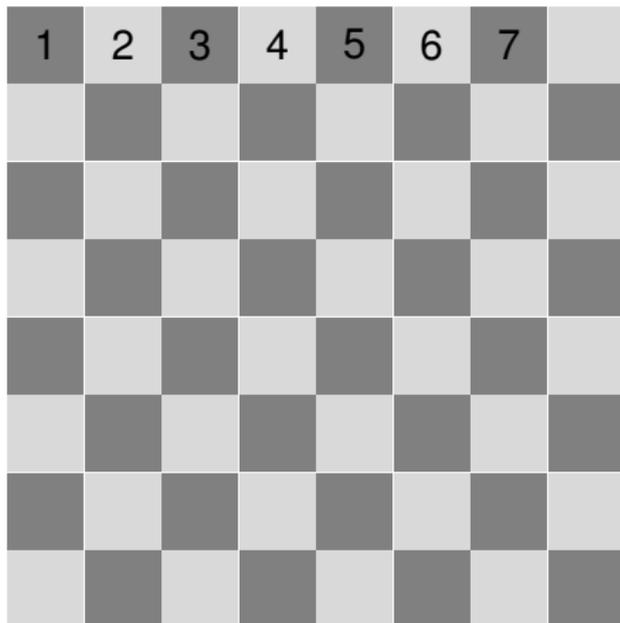
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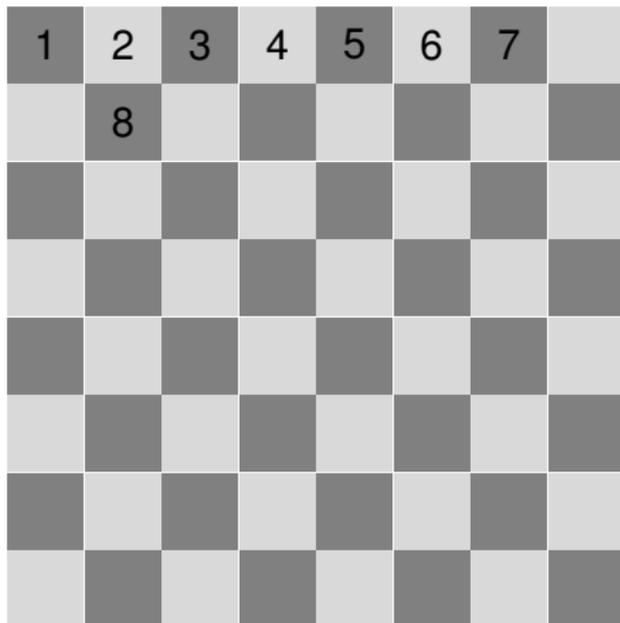
Ocho reinas, peor algoritmo posible



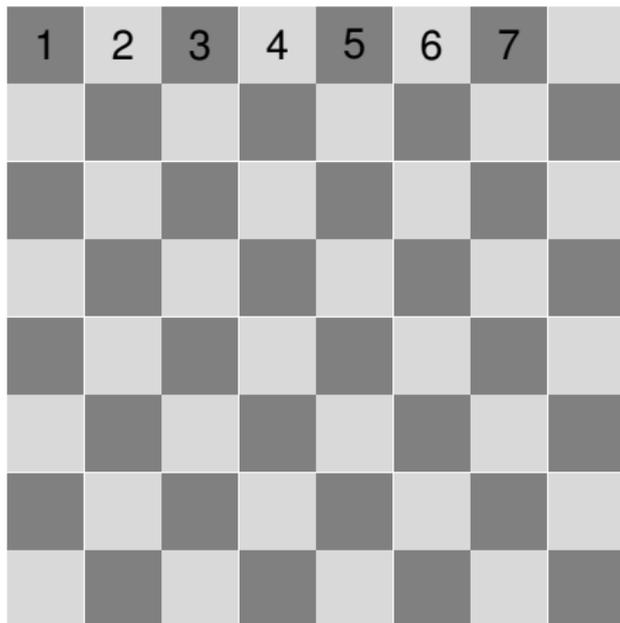
Ocho reinas, peor algoritmo posible



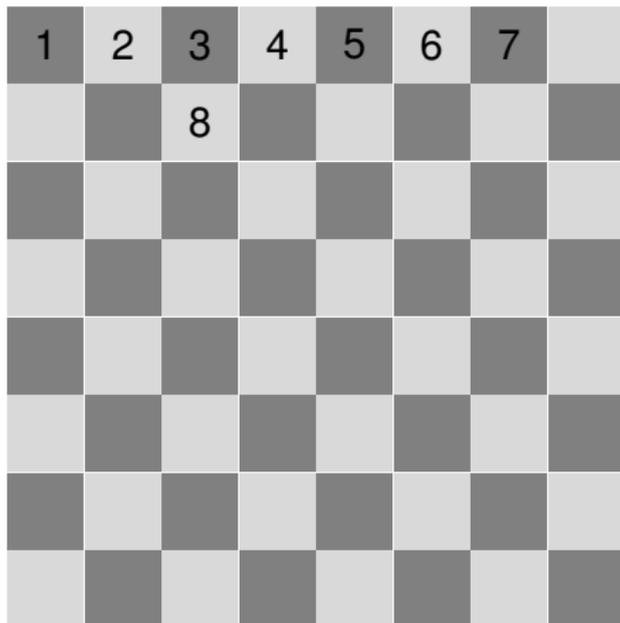
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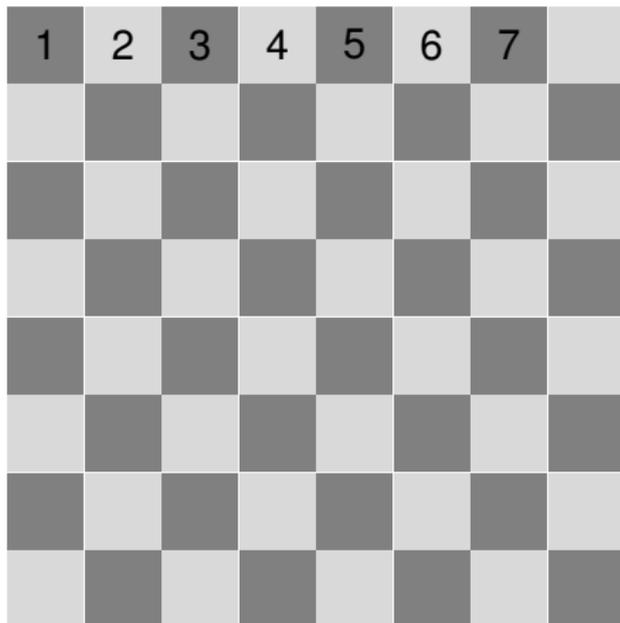
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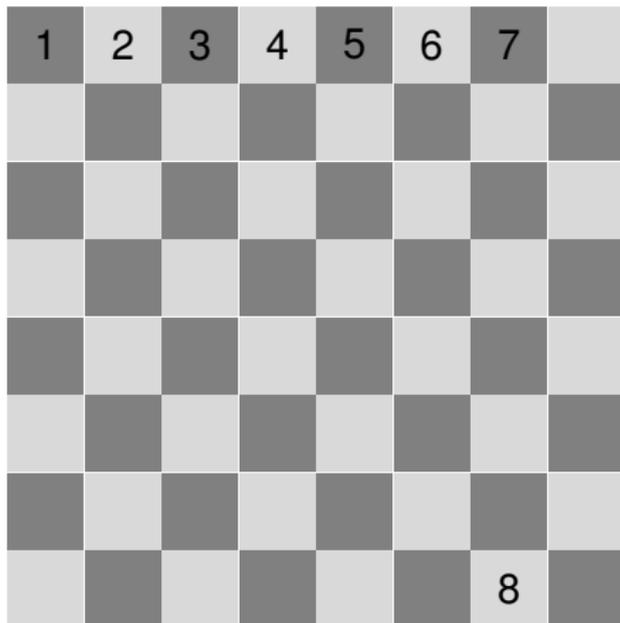
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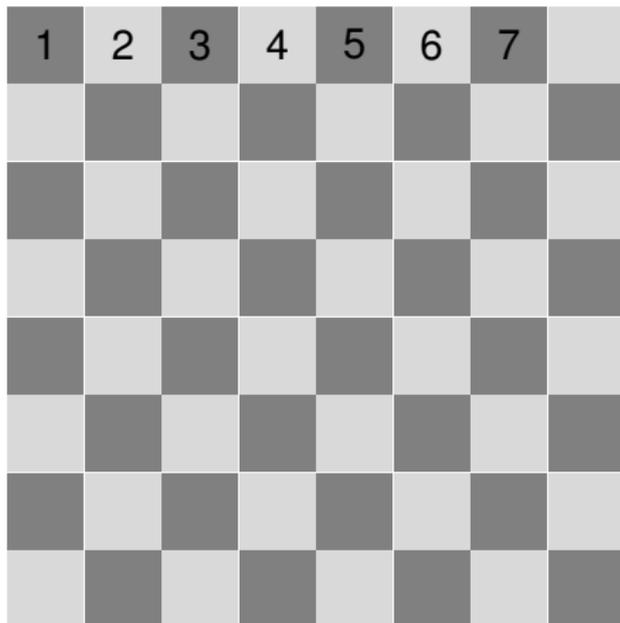
Ocho reinas, peor algoritmo posible



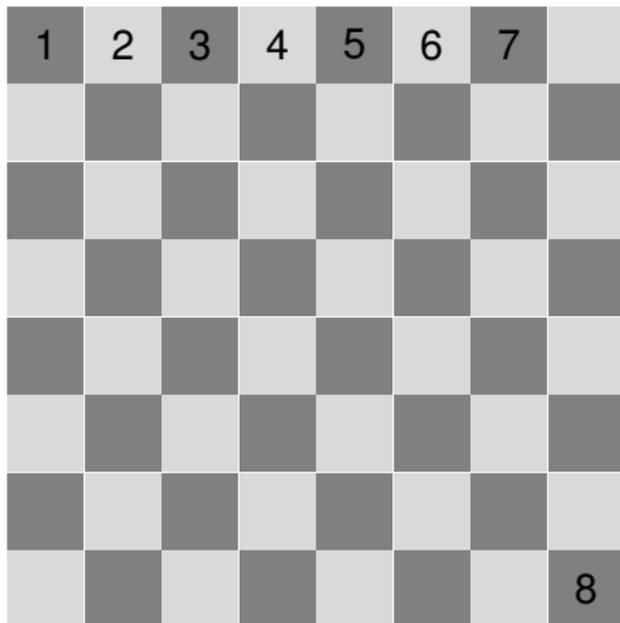
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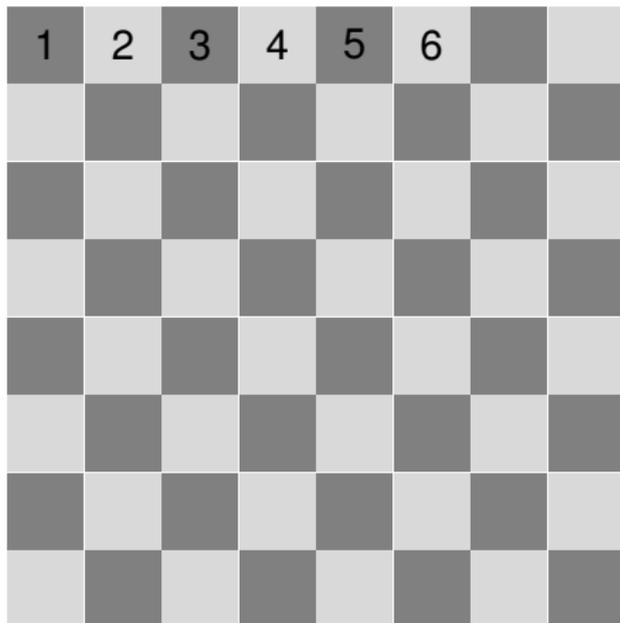
Ocho reinas, peor algoritmo posible



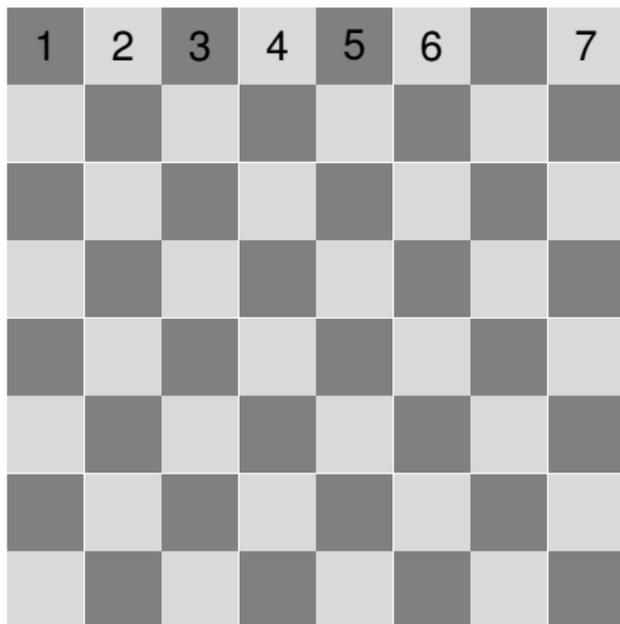
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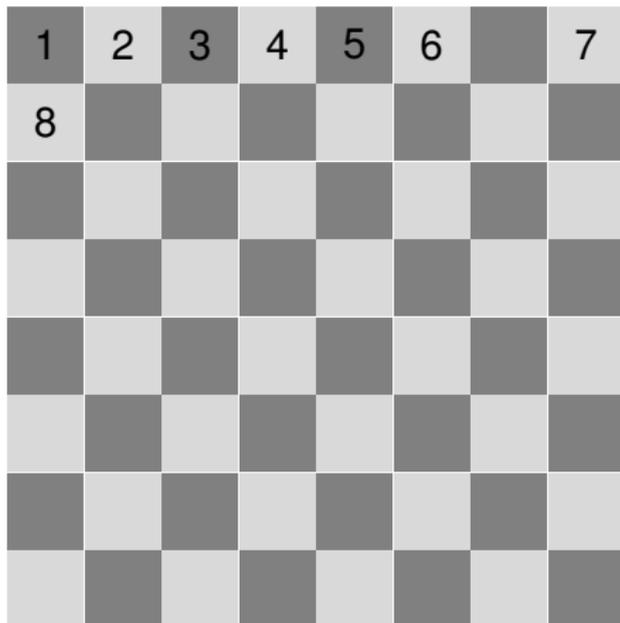
Ocho reinas, peor algoritmo posible



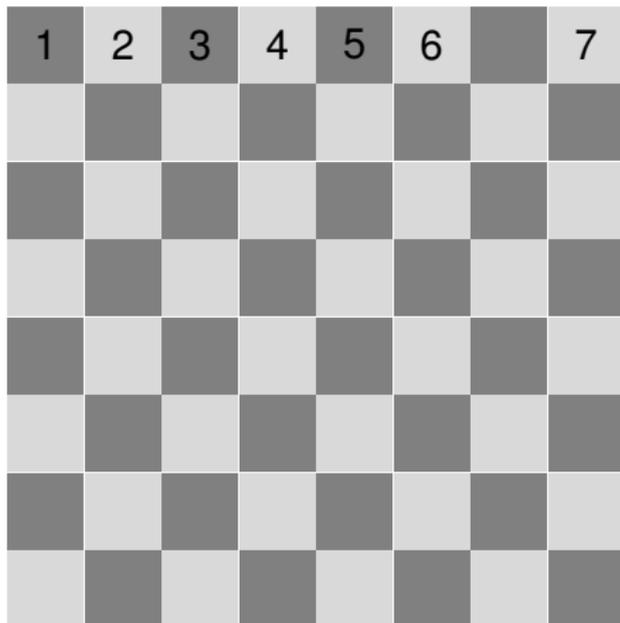
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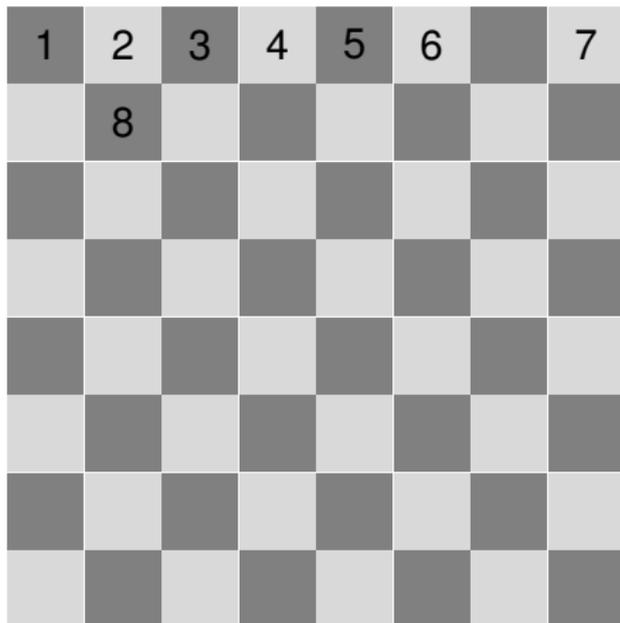
Ocho reinas, peor algoritmo posible



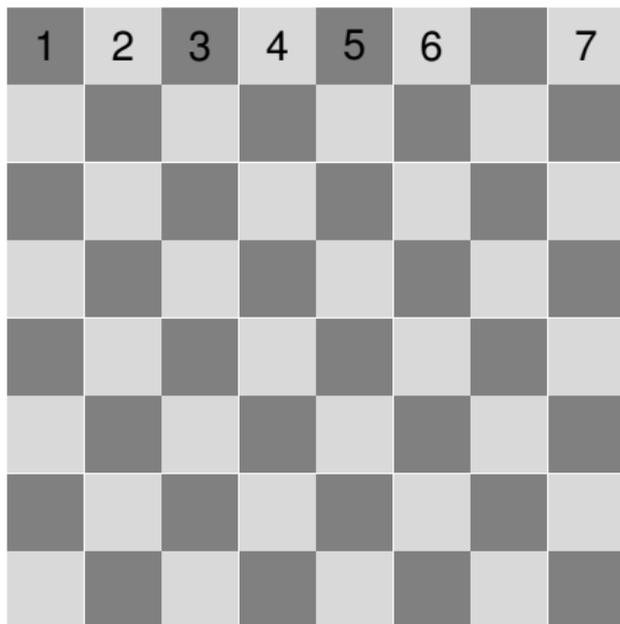
Ocho reinas, peor algoritmo posible



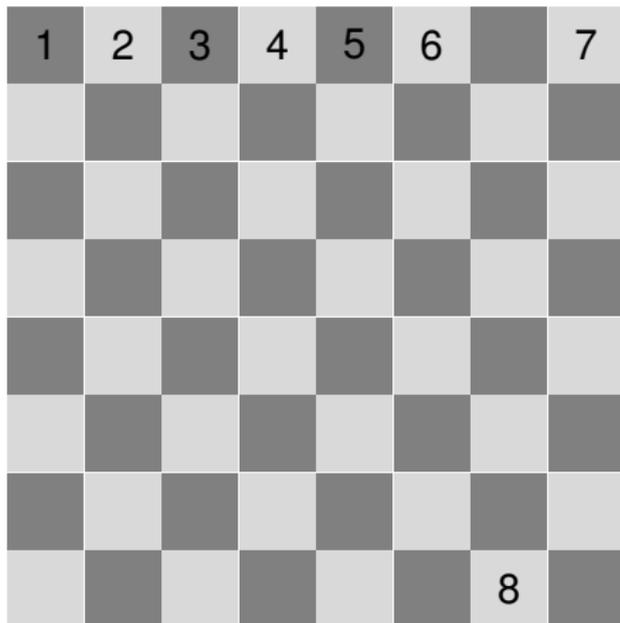
Ocho reinas, peor algoritmo posible



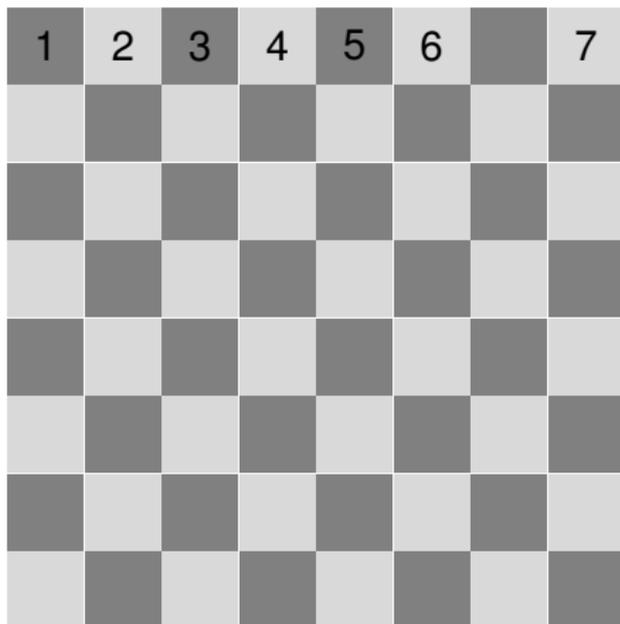
Ocho reinas, peor algoritmo posible



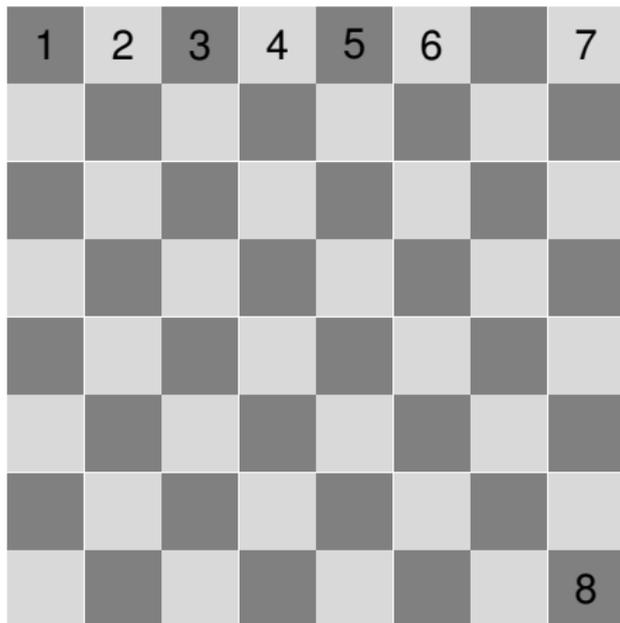
Ocho reinas, peor algoritmo posible



Ocho reinas, peor algoritmo posible



Ocho reinas, peor algoritmo posible



Ocho reinas, peor algoritmo posible

El algoritmo

Calcula el número de maneras de ubicar 8 reinas sin que se amenacen.

```

fun ocho_reinas_1() ret r: nat
  r:= 0
  for i1:= 1 to 57 do
    for i2:= i1+1 to 58 do
      ...
      for i8:= i7+1 to 64 do
        if solucion_1([i1,i2,i3,i4,i5,i6,i7,i8]) then r:= r+1 fi
      od
      ...
    od
  od
end

```

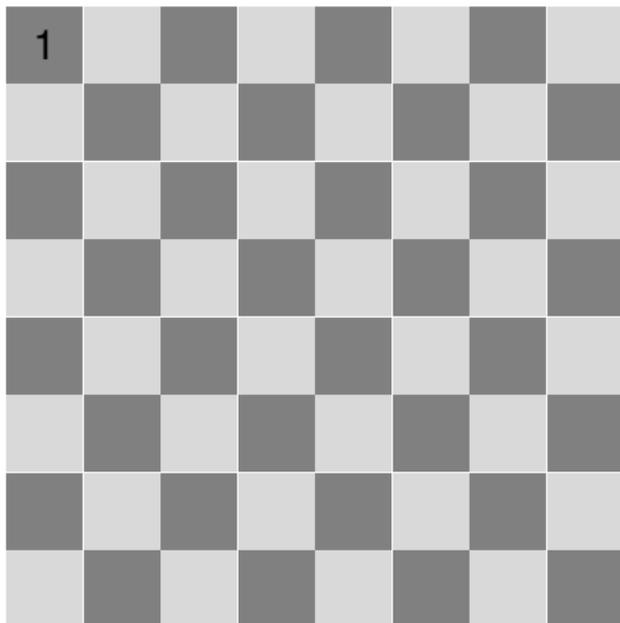
Ocho reinas, peor algoritmo posible

El grafo implícito

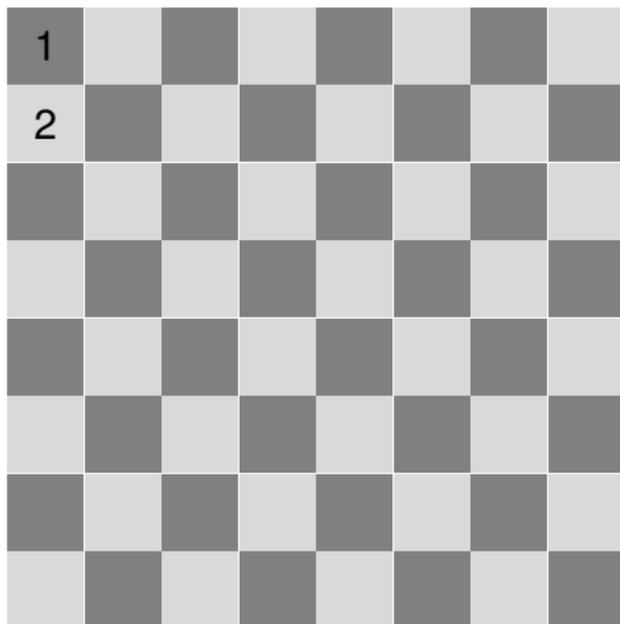
$$V = \{[p_1, p_2, \dots, p_n] \in \{1, \dots, 64\}^* \mid \\ n \leq 8 \wedge p_1 < p_2 < \dots < p_n \leq 56 + n\}$$

Dados $p = [p_1, p_2, \dots, p_n] \in V$ y $q = [q_1, q_2, \dots, q_m] \in V$ hay una arista de p a q sii $m = n + 1$ y $p_i = q_i$ para todo $1 \leq i \leq n$.

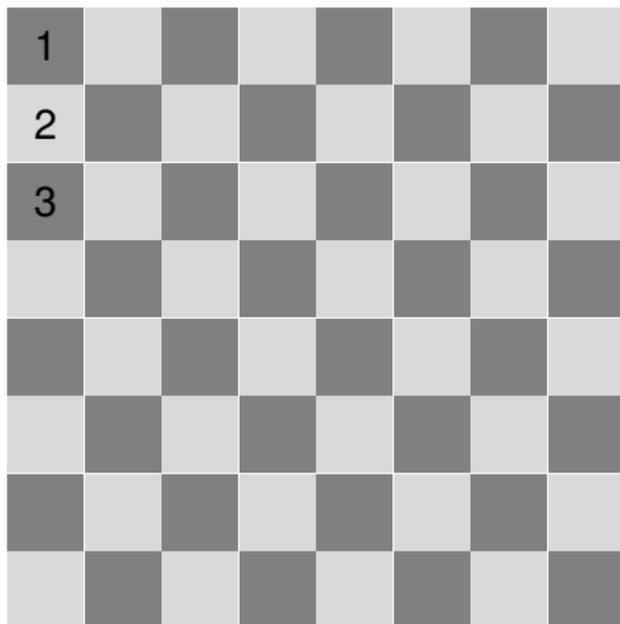
Ocho reinas, un algoritmo menos malo



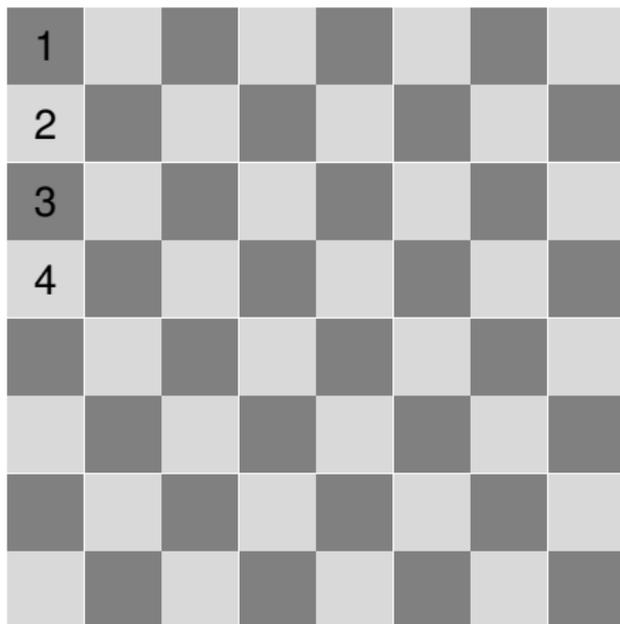
Ocho reinas, un algoritmo menos malo



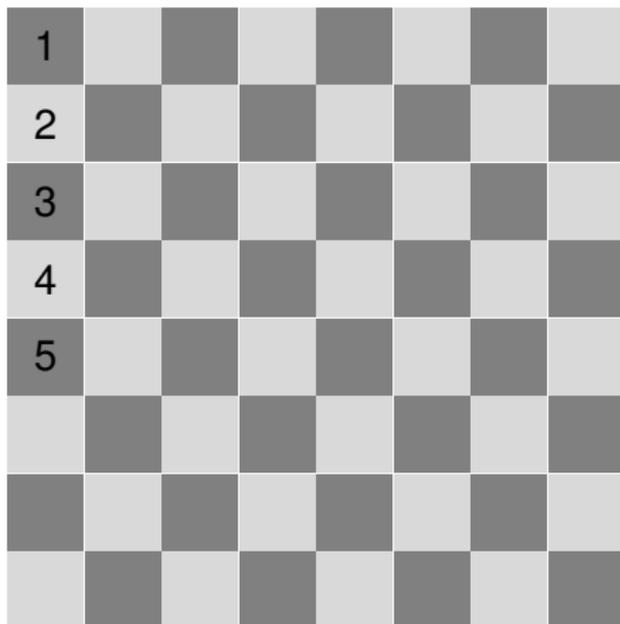
Ocho reinas, un algoritmo menos malo



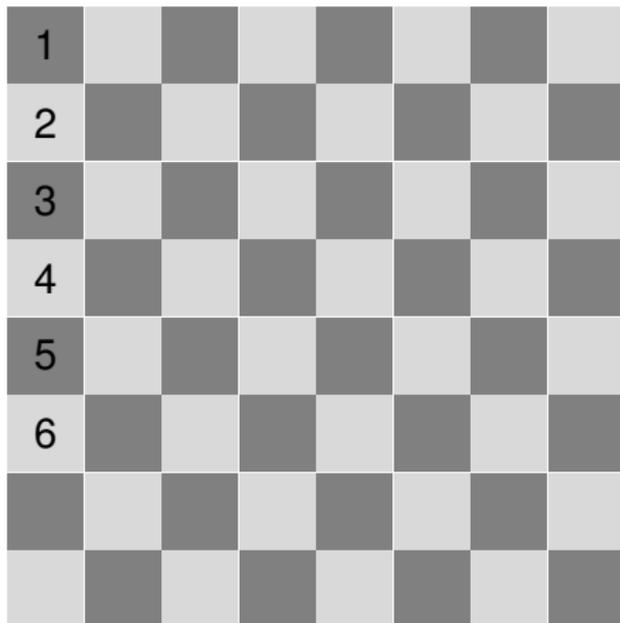
Ocho reinas, un algoritmo menos malo



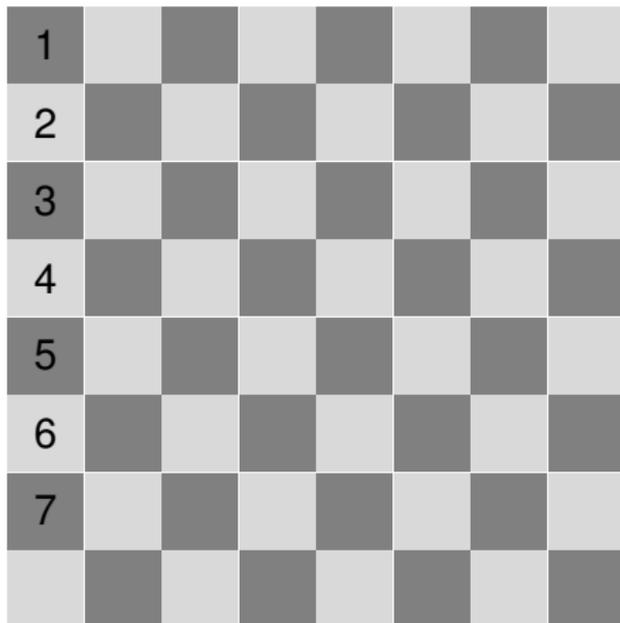
Ocho reinas, un algoritmo menos malo



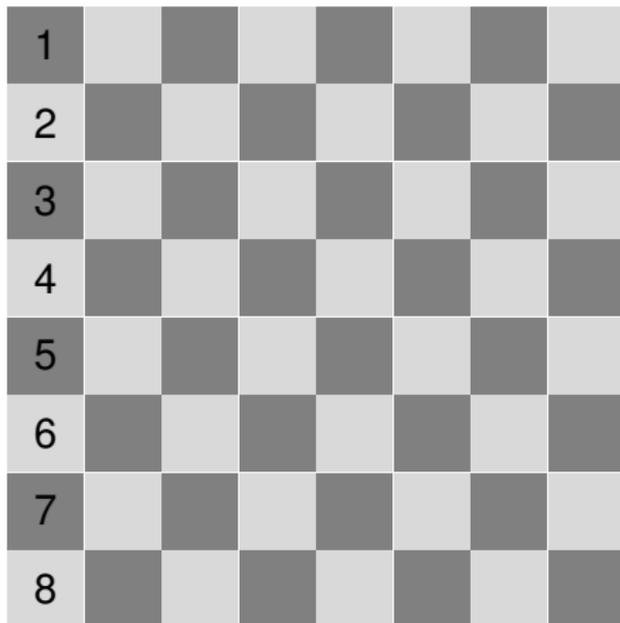
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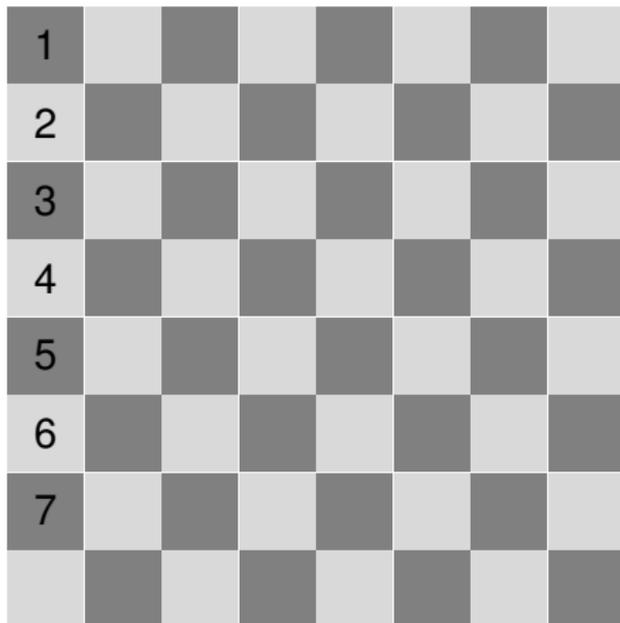
Ocho reinas, un algoritmo menos malo



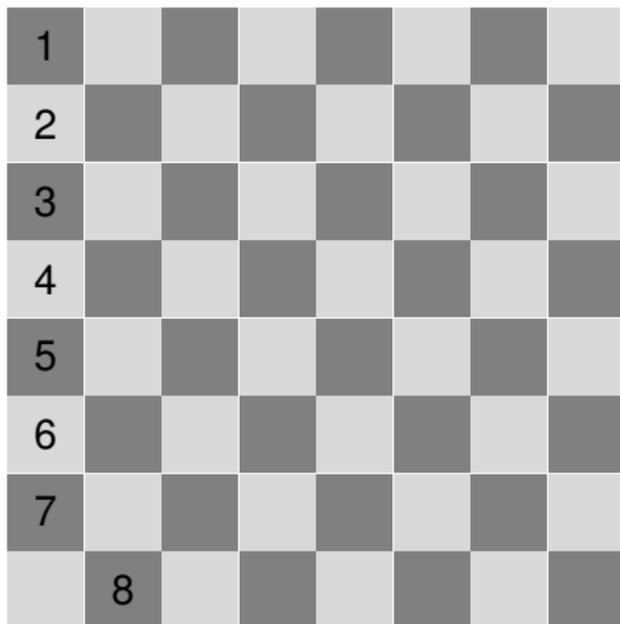
Ocho reinas, un algoritmo menos malo



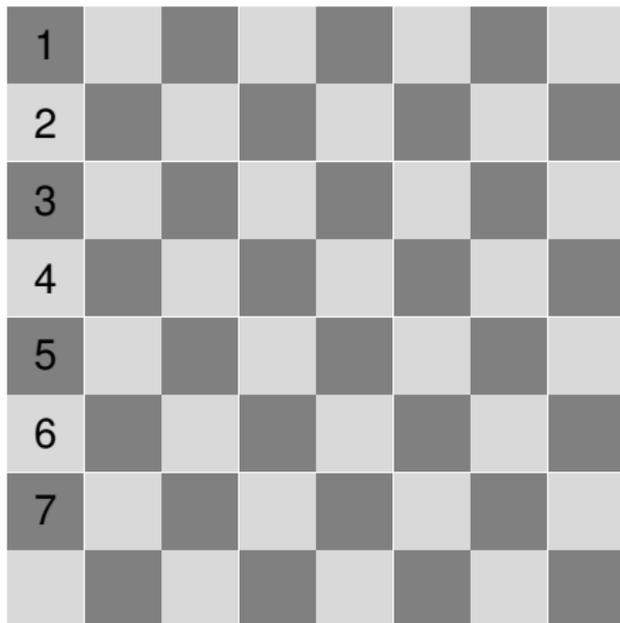
Ocho reinas, un algoritmo menos malo



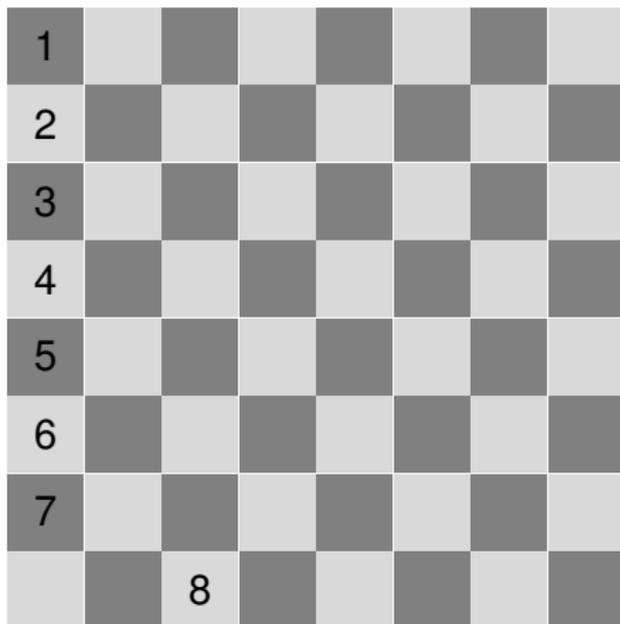
Ocho reinas, un algoritmo menos malo



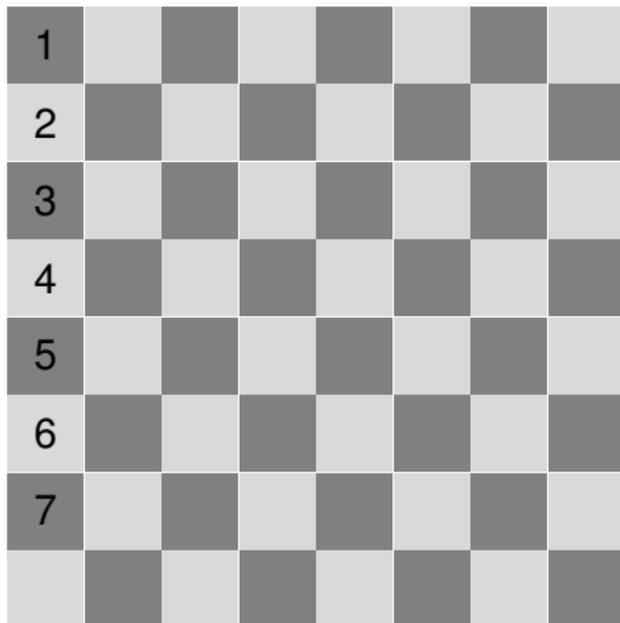
Ocho reinas, un algoritmo menos malo



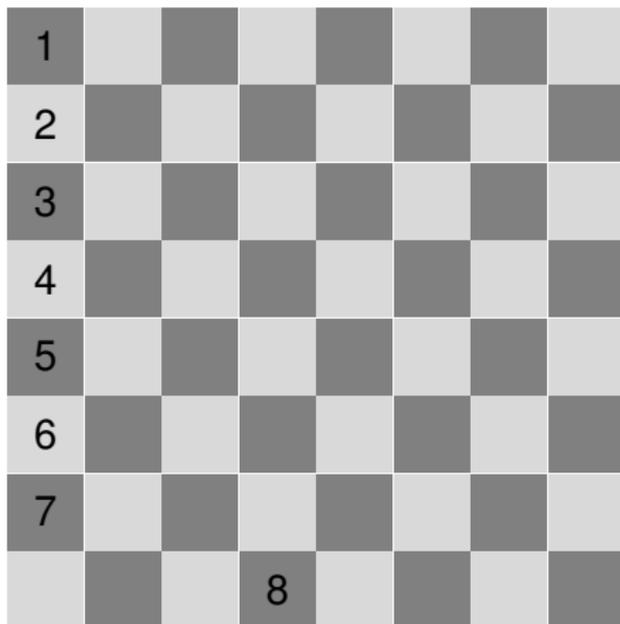
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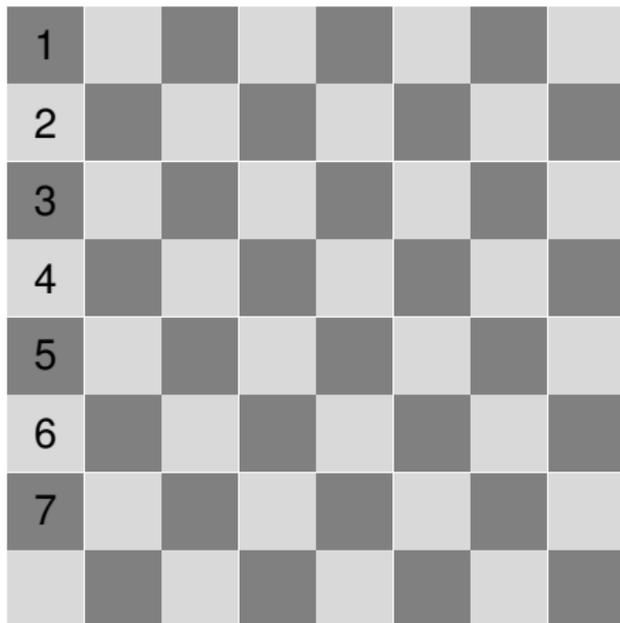
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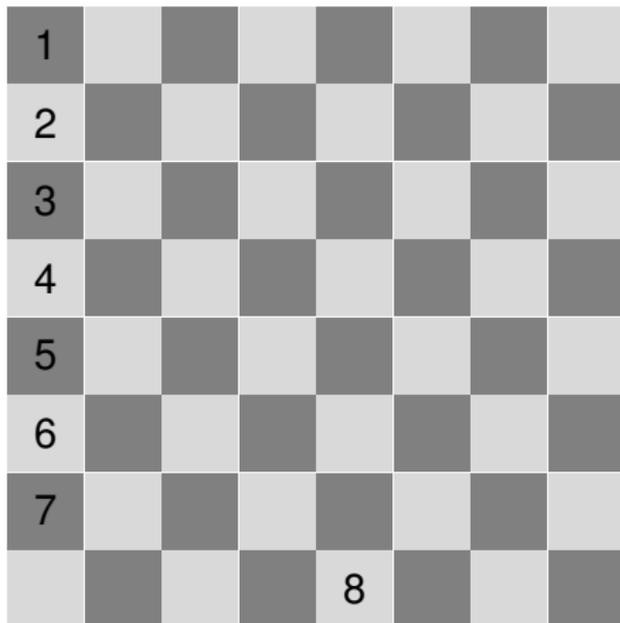
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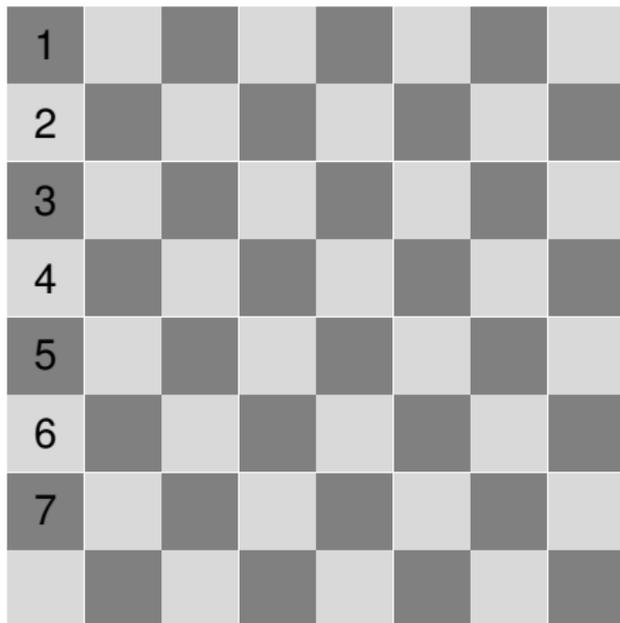
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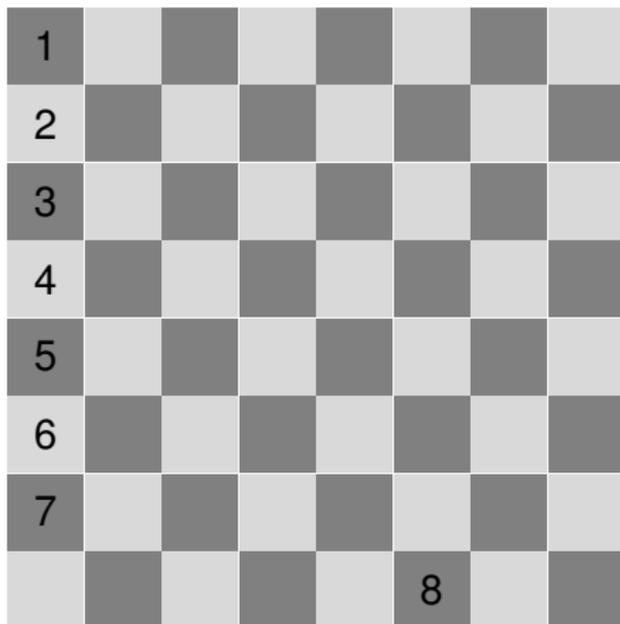
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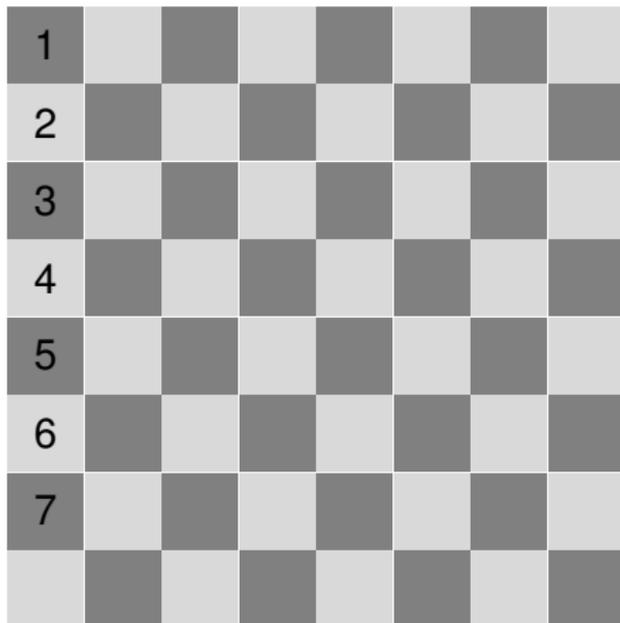
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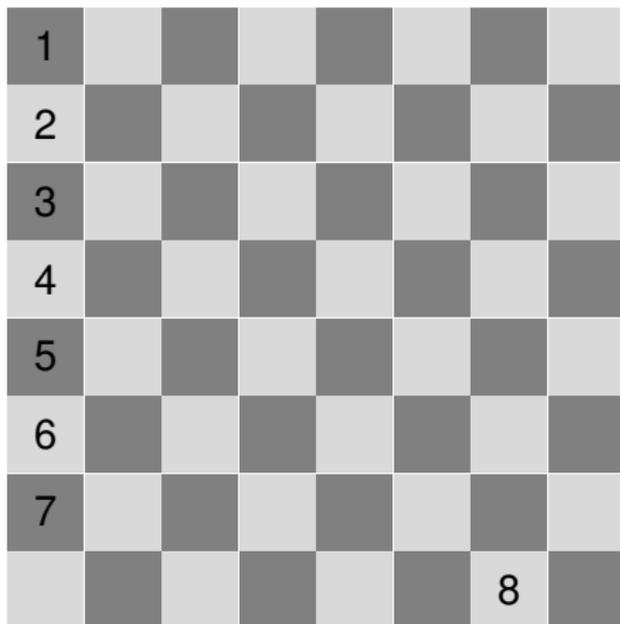
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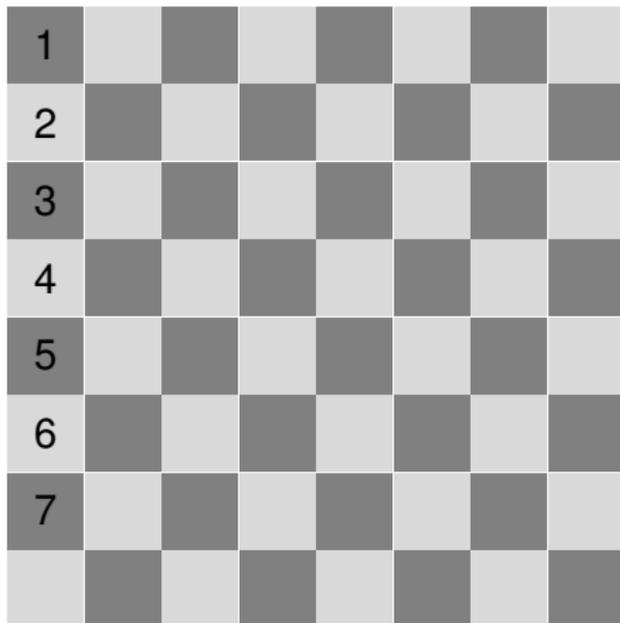
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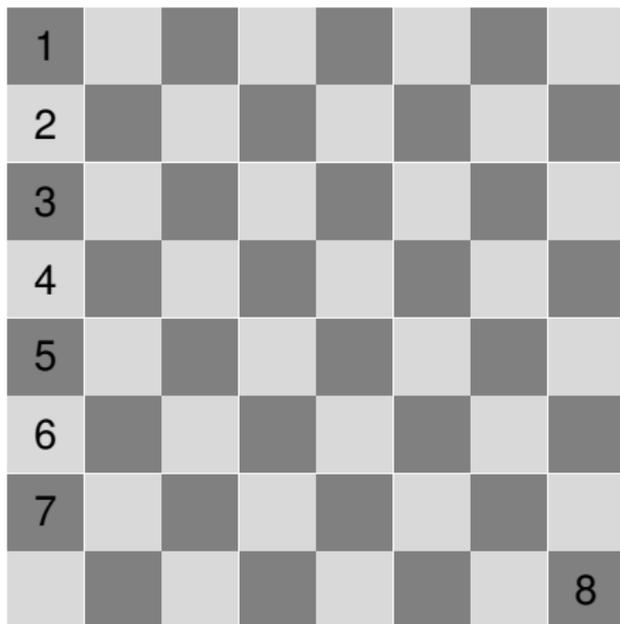
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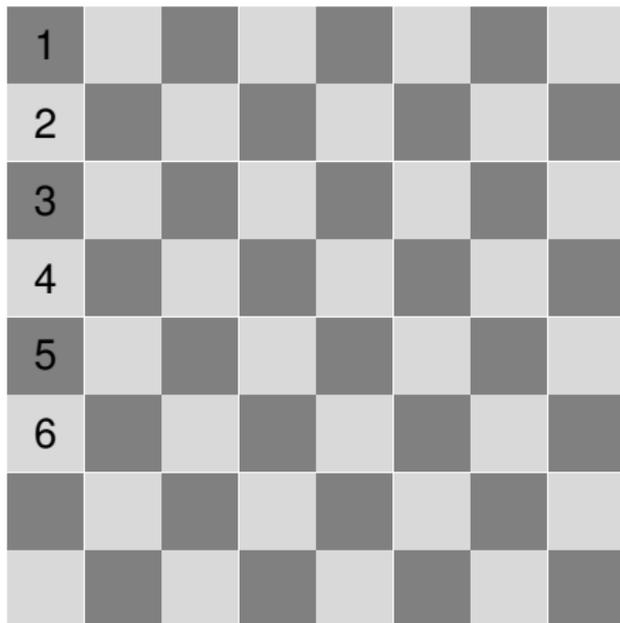
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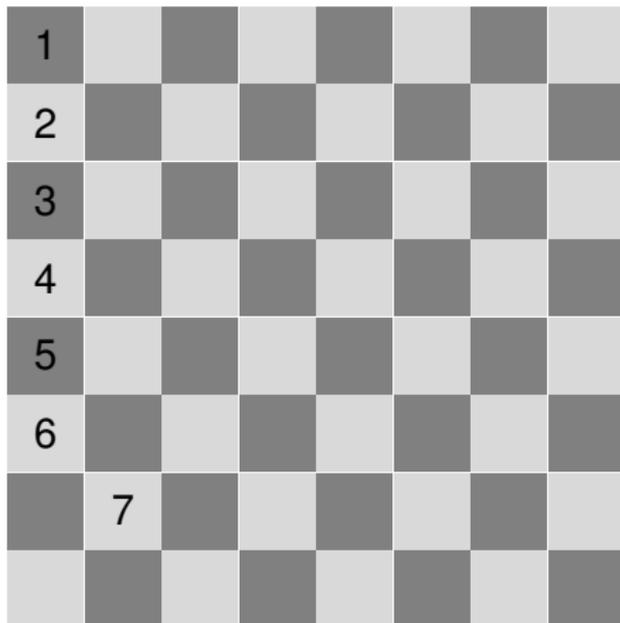
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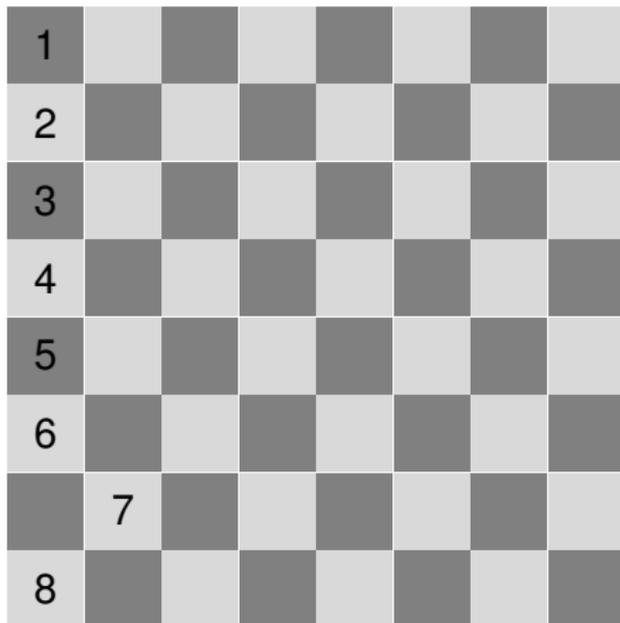
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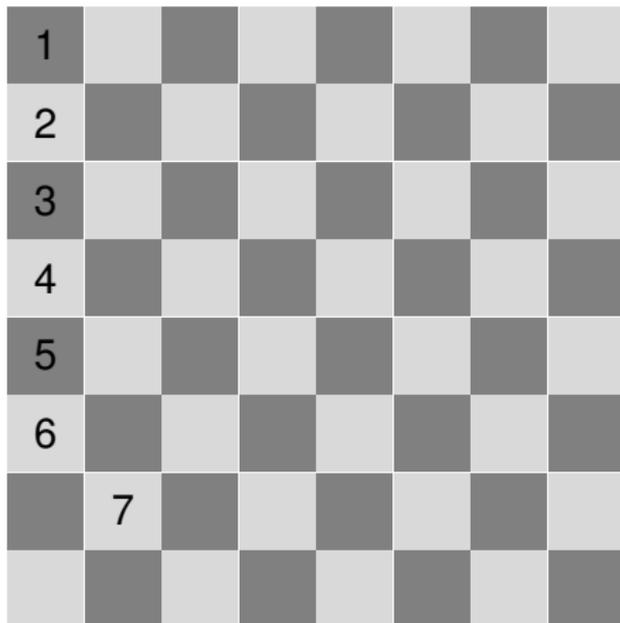
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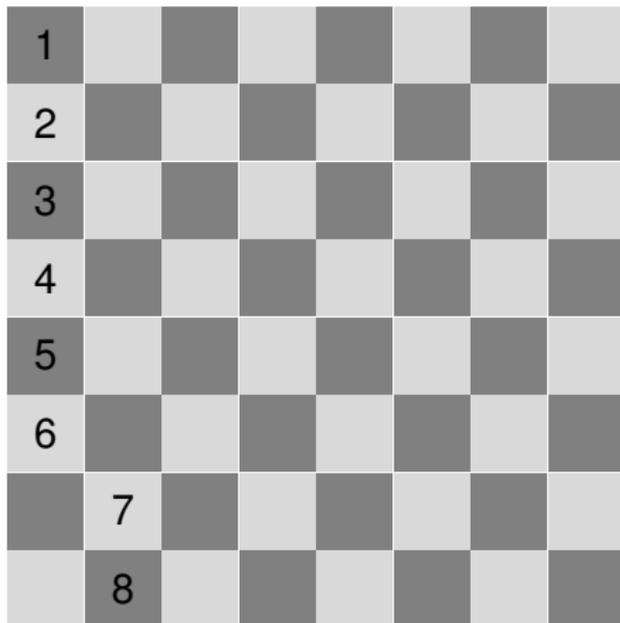
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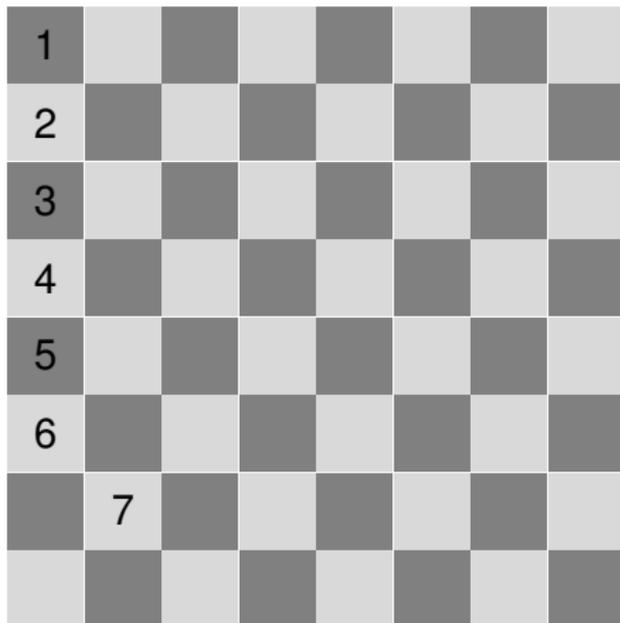
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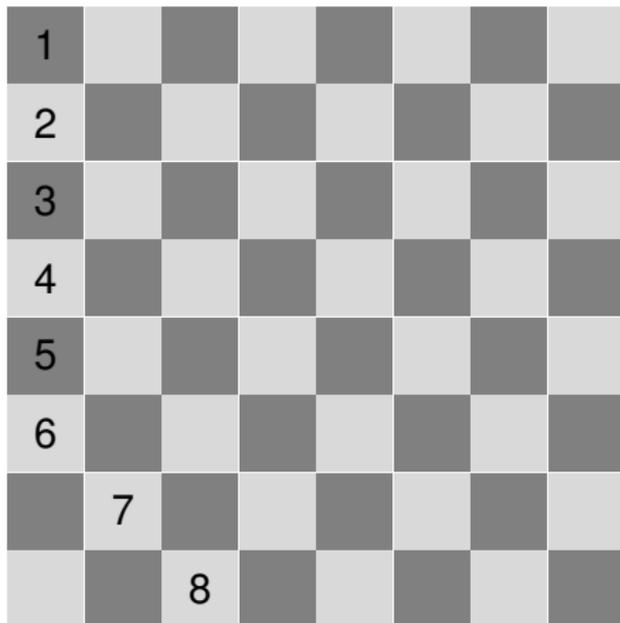
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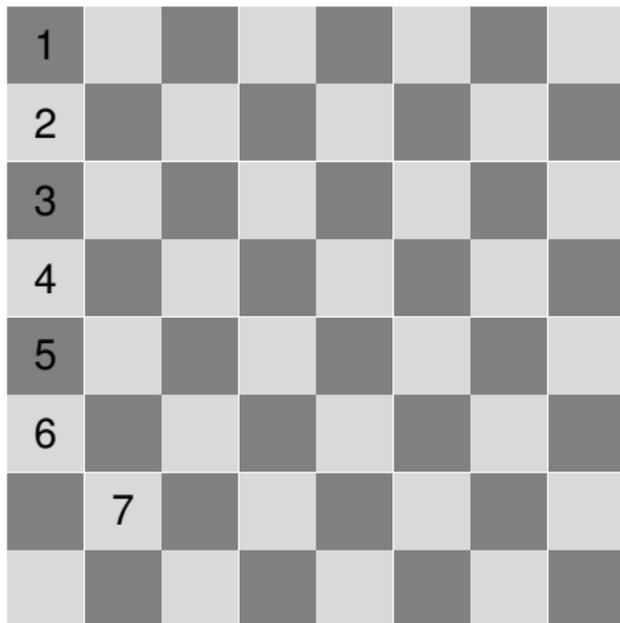
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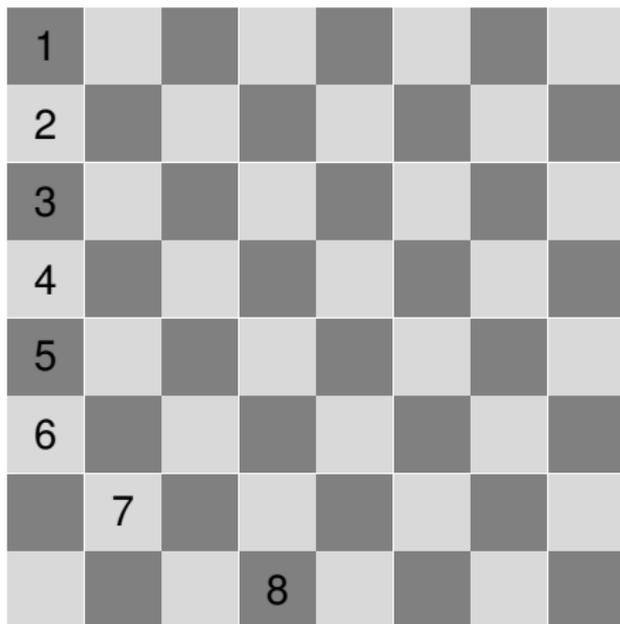
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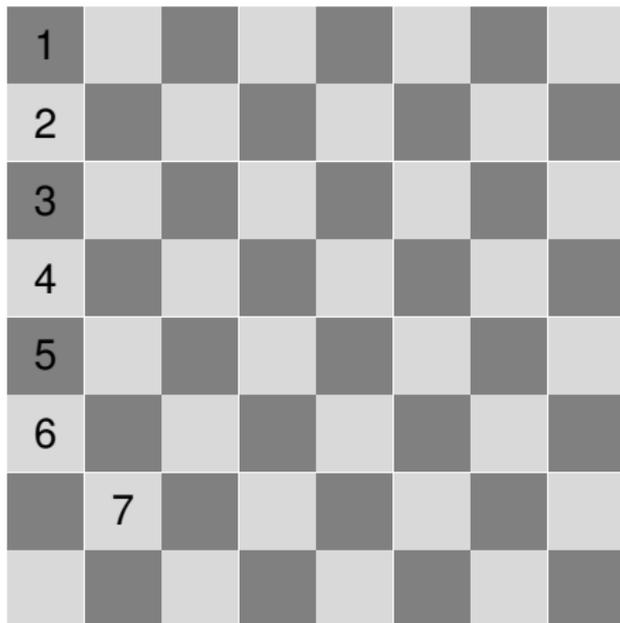
Ocho reinas, un algoritmo menos malo



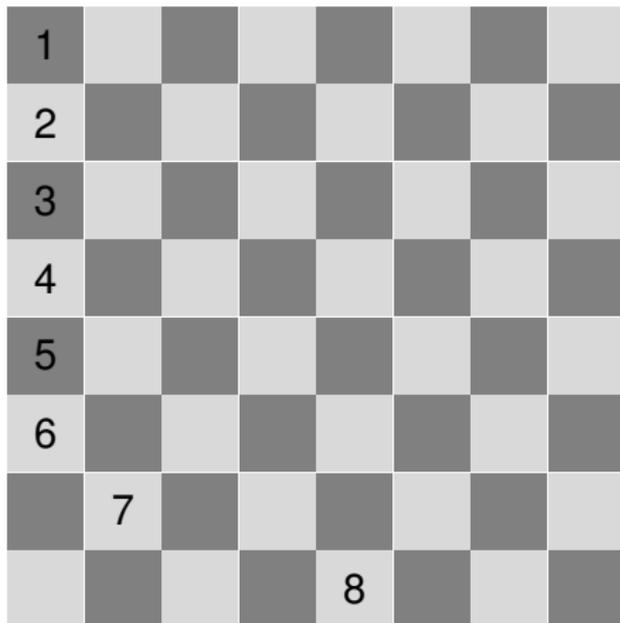
Ocho reinas, un algoritmo menos malo



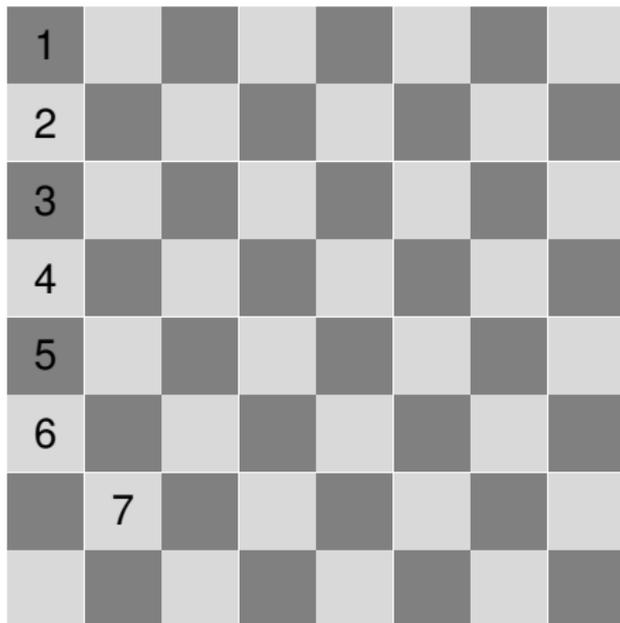
Ocho reinas, un algoritmo menos malo



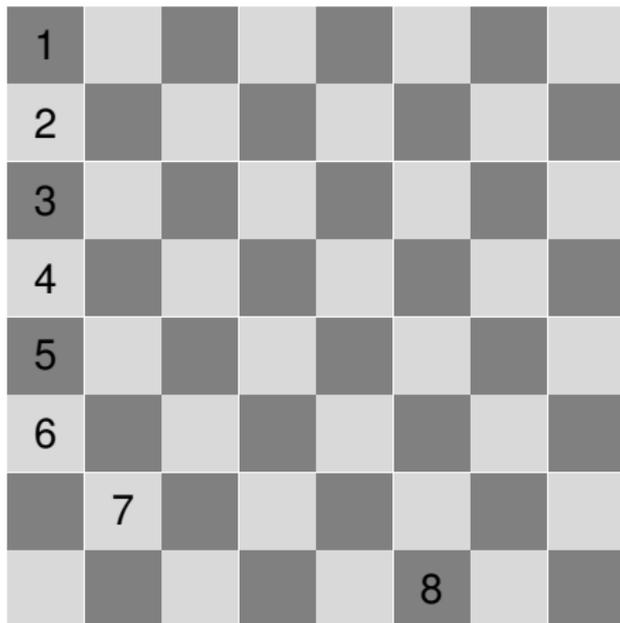
Ocho reinas, un algoritmo menos malo



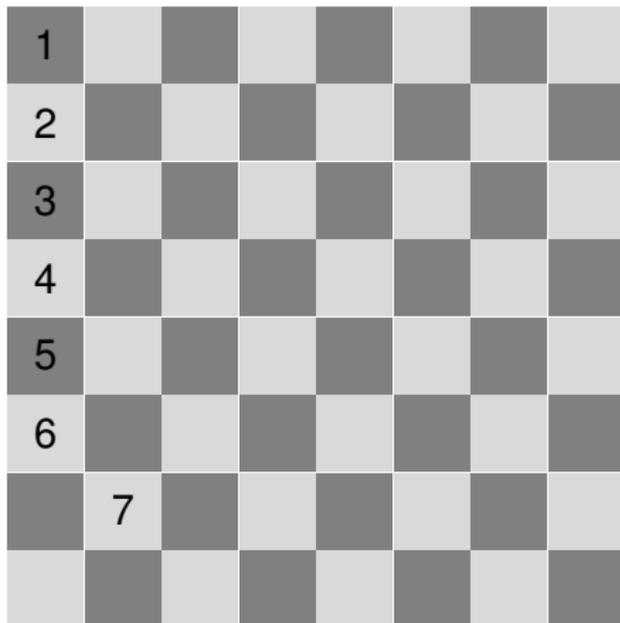
Ocho reinas, un algoritmo menos malo



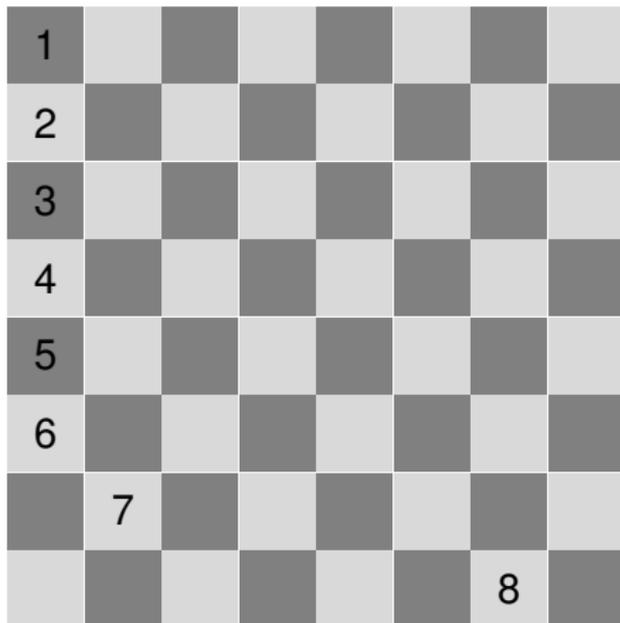
Ocho reinas, un algoritmo menos malo



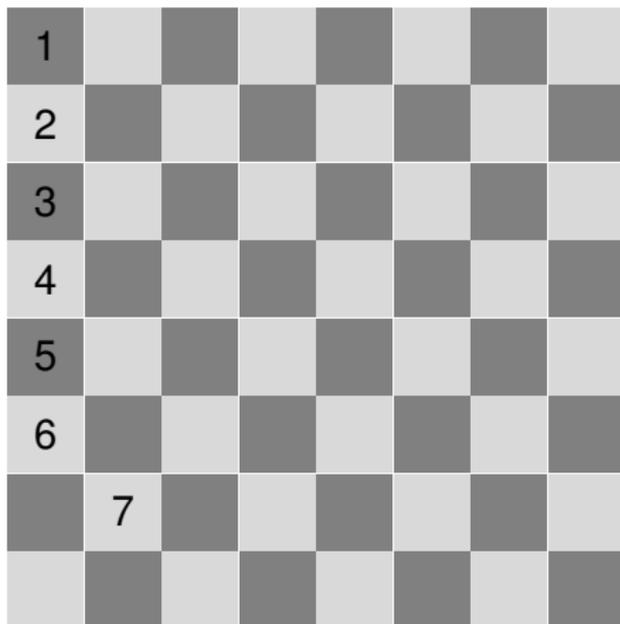
Ocho reinas, un algoritmo menos malo



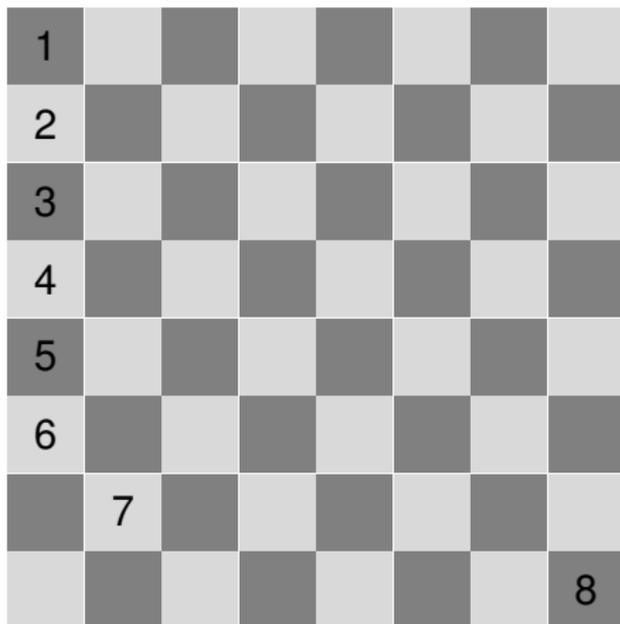
Ocho reinas, un algoritmo menos malo



Ocho reinas, un algoritmo menos malo



Ocho reinas, un algoritmo menos malo



Ocho reinas, un algoritmo menos malo

El algoritmo

Calcula el número de maneras de ubicar 8 reinas sin que se amenacen.

```

fun ocho_reinas_2() ret r: nat
  r:= 0
  for j1:= 1 to 8 do
    for j2:= 1 to 8 do
      ...
      for j8:= 1 to 8 do
        if solucion_2([j1,j2,j3,j4,j5,j6,j7,j8]) then r:= r+1 fi
      od
      ...
    od
  od
end

```

Ocho reinas, un algoritmo menos malo

El grafo implícito

$$V = \{p \in \{1, \dots, 8\}^* \mid |p| \leq 8\}$$

Y las aristas se definen como antes.

Ocho reinas, versión recursiva

```
fun ocho_reinas_2() ret r: nat
```

```
  r:= 0
```

```
  or_2([ ], r)
```

```
end
```

```
proc or_2(in sol: list of nat, in/out r: nat)
```

```
  {calcula el número de maneras de extender sol}
```

```
  {hasta ubicar en total 8 reinas sin que se amenacen}
```

```
if |sol| = 8 then
```

```
  if solucion_2(sol) then r:= r+1 fi
```

```
else for j:= 1 to 8 do
```

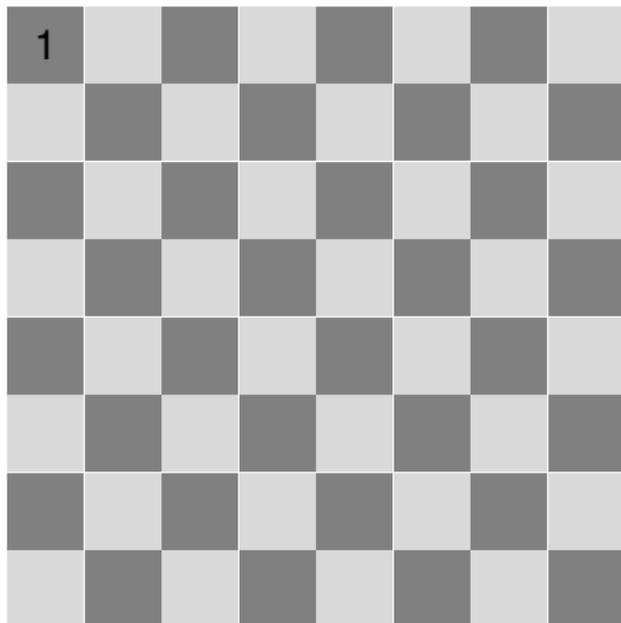
```
  or_2(sol < j, r)
```

```
od
```

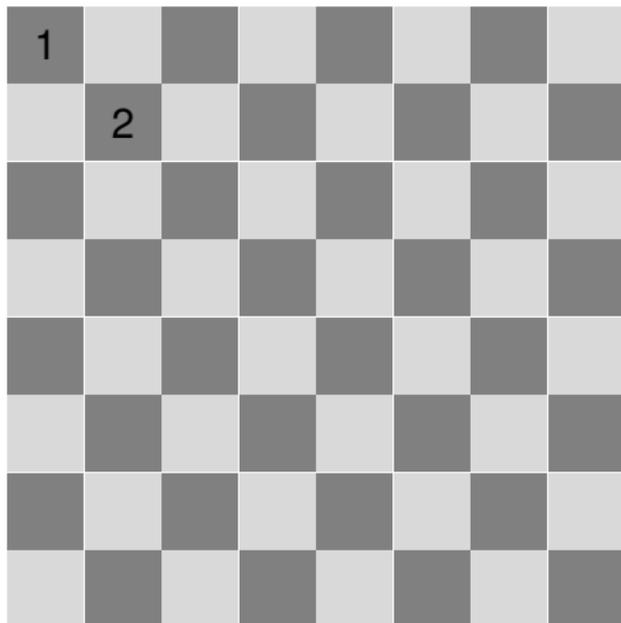
```
fi
```

```
end
```

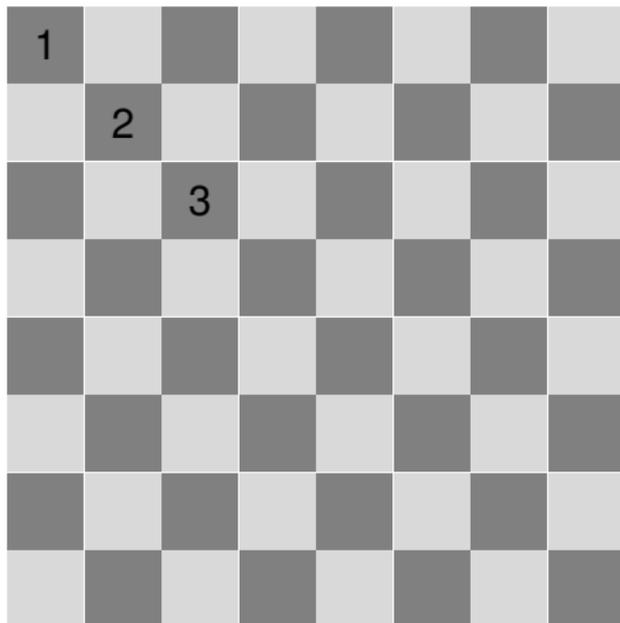
Ocho reinas, un algoritmo mejor



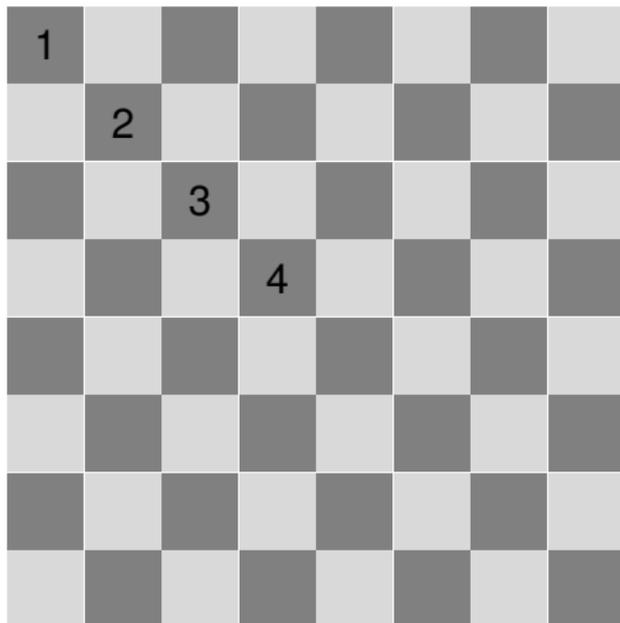
Ocho reinas, un algoritmo mejor



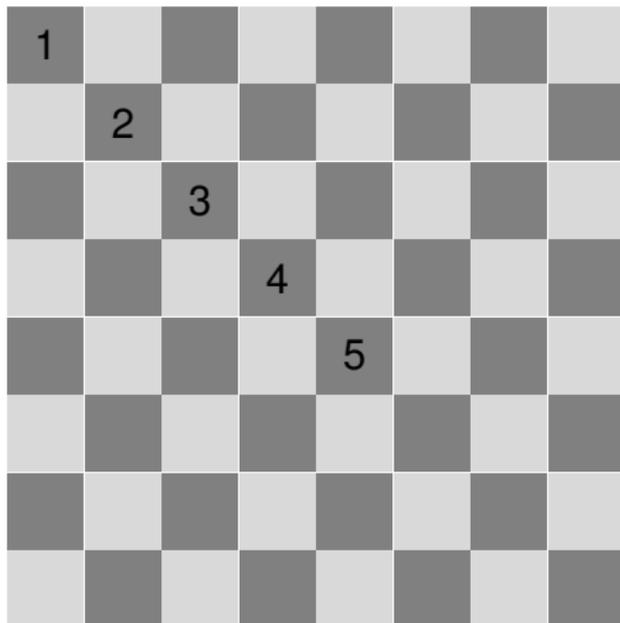
Ocho reinas, un algoritmo mejor



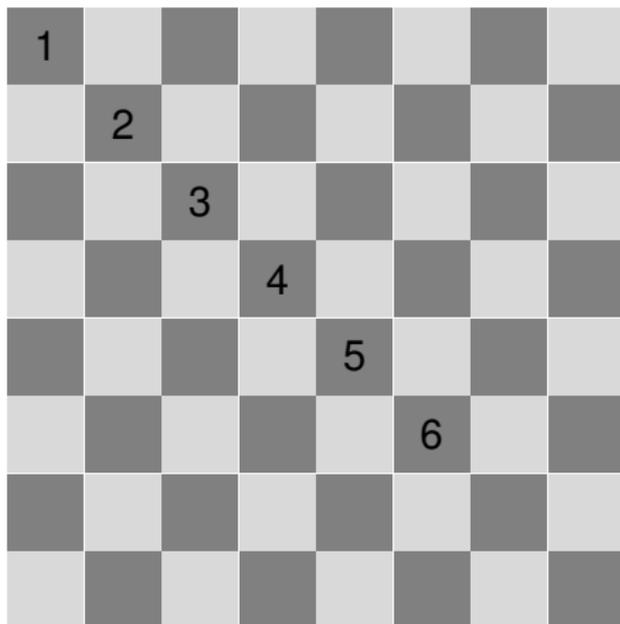
Ocho reinas, un algoritmo mejor



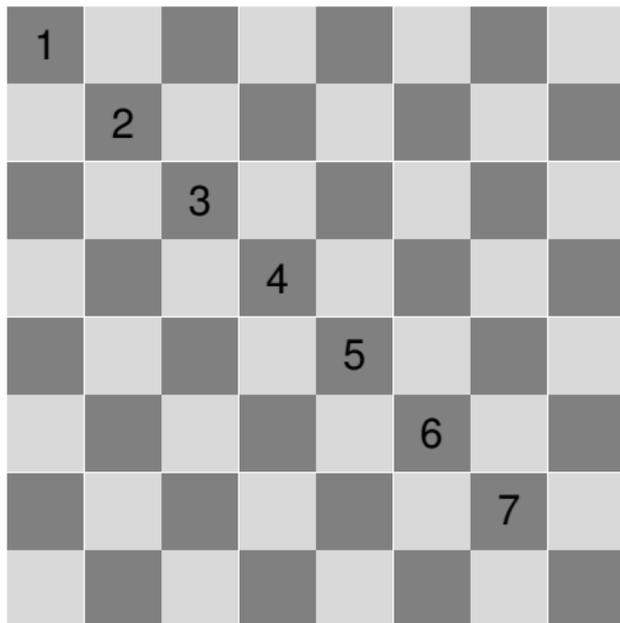
Ocho reinas, un algoritmo mejor



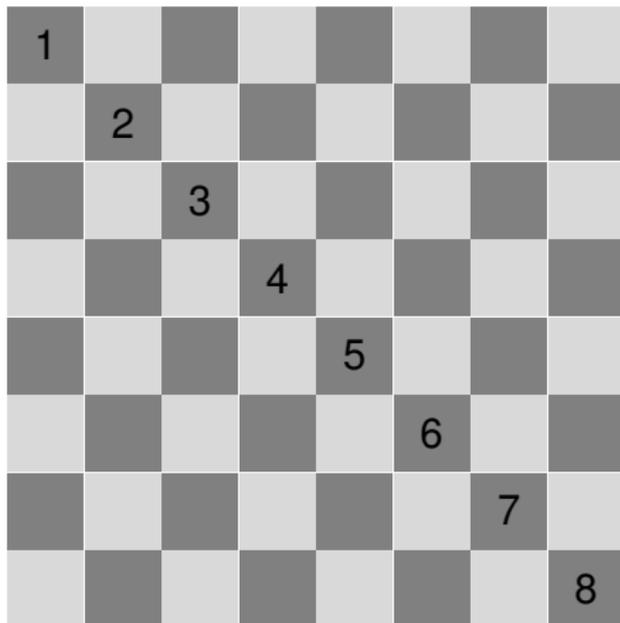
Ocho reinas, un algoritmo mejor



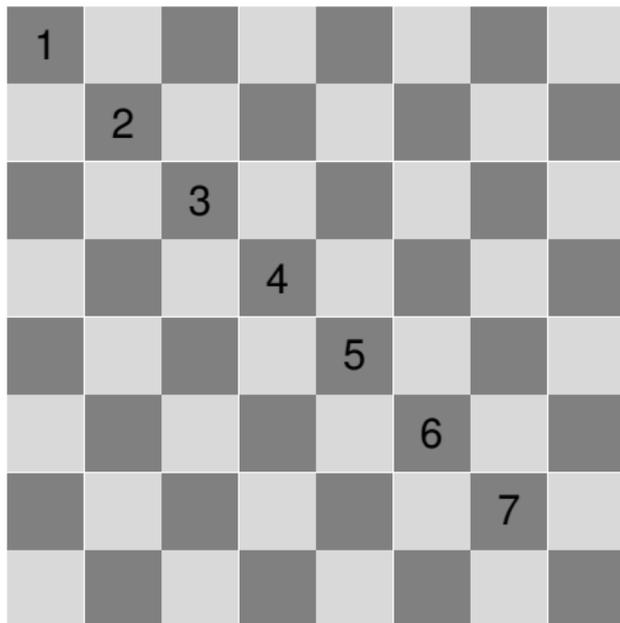
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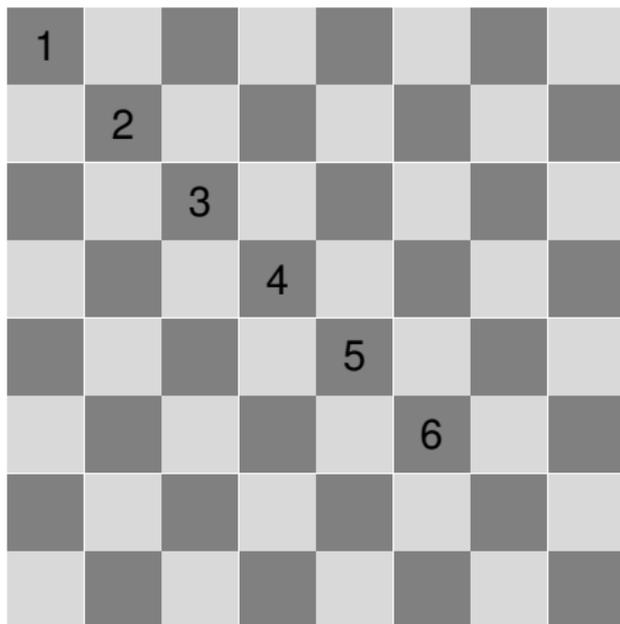
Ocho reinas, un algoritmo mejor



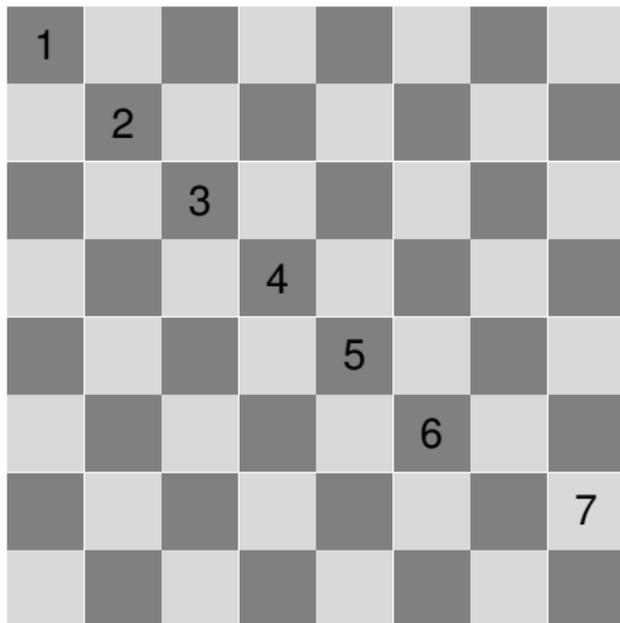
Ocho reinas, un algoritmo mejor



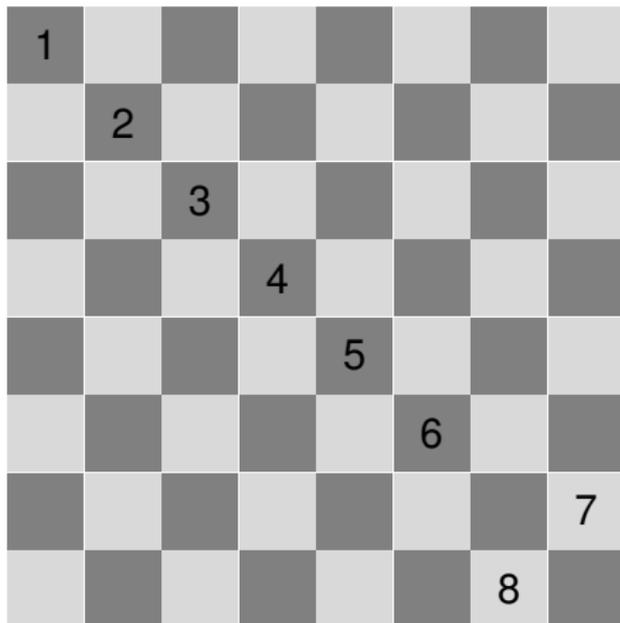
Ocho reinas, un algoritmo mejor



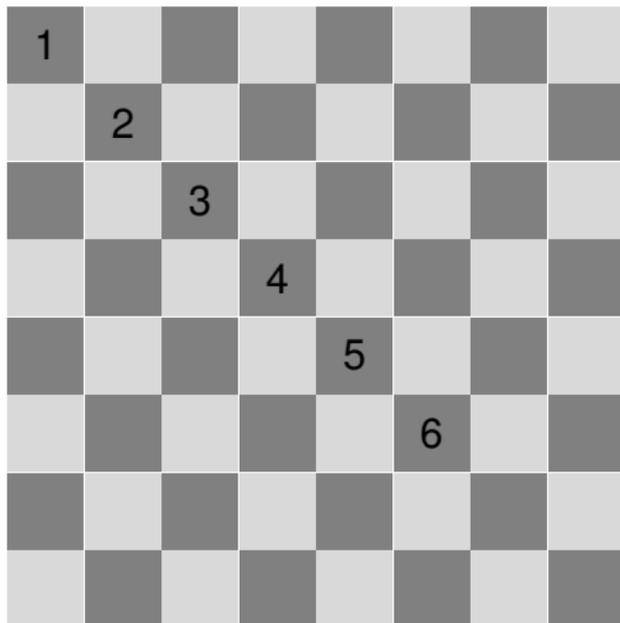
Ocho reinas, un algoritmo mejor



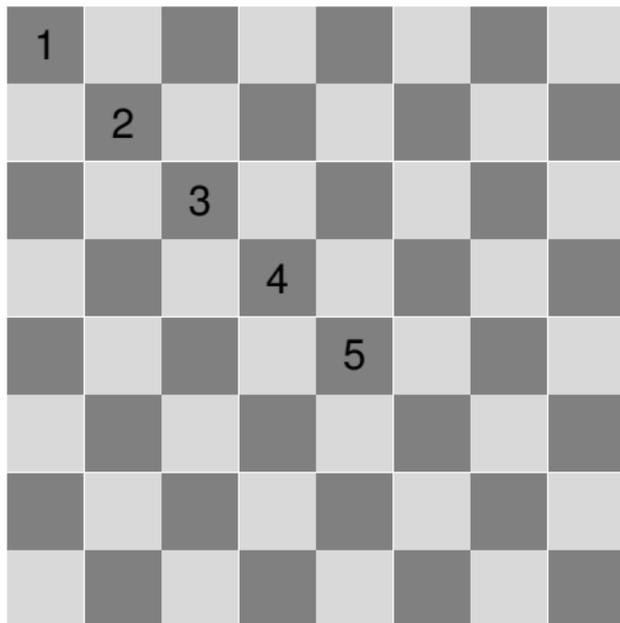
Ocho reinas, un algoritmo mejor



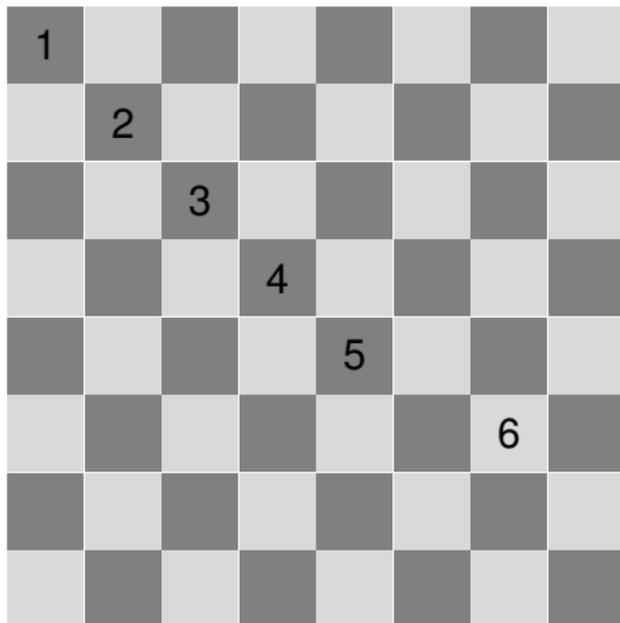
Ocho reinas, un algoritmo mejor



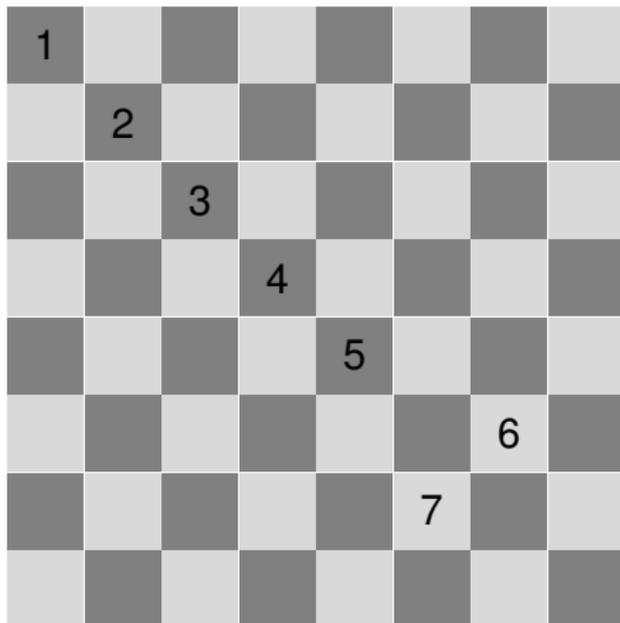
Ocho reinas, un algoritmo mejor



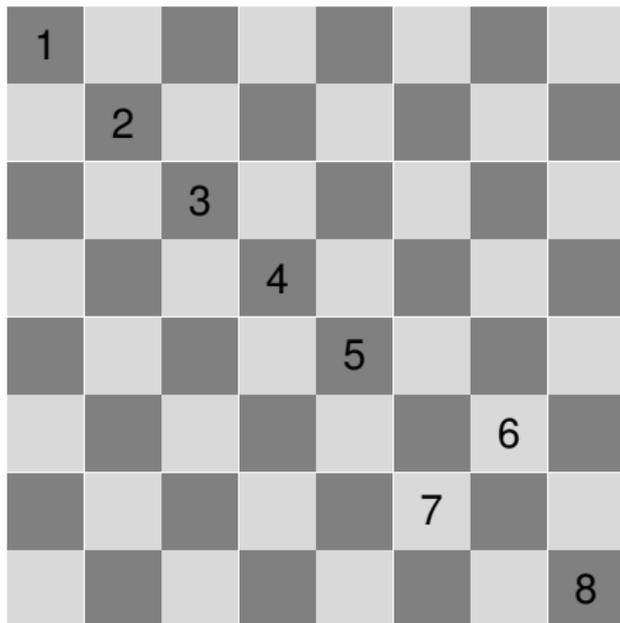
Ocho reinas, un algoritmo mejor



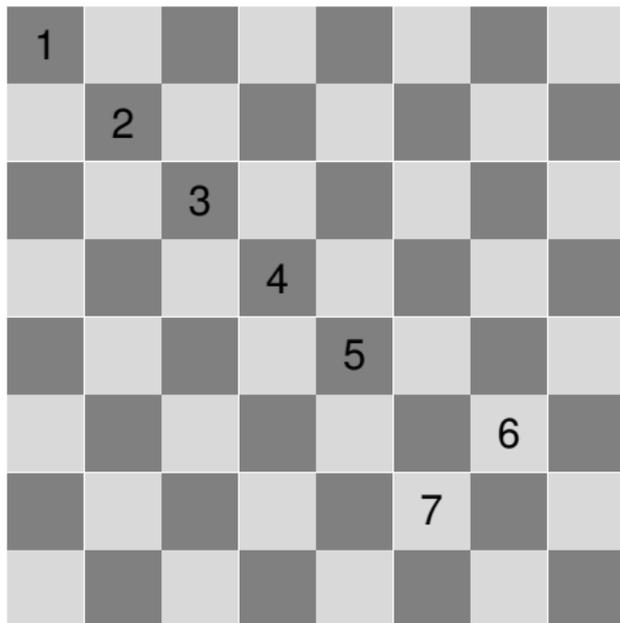
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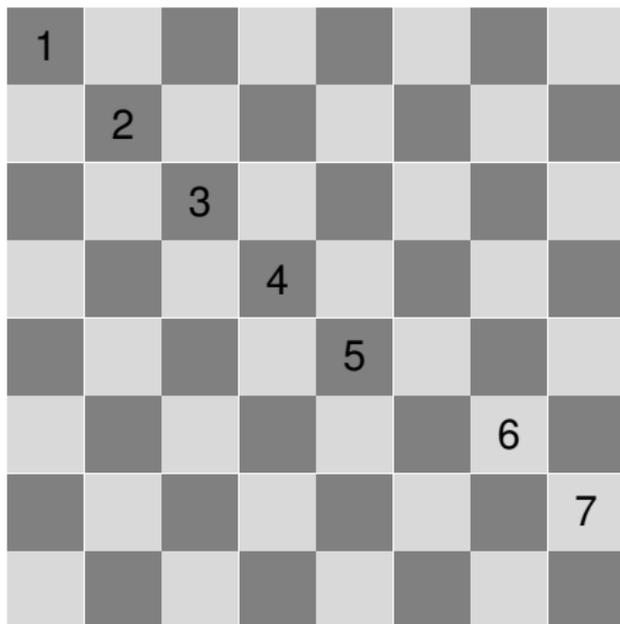
Ocho reinas, un algoritmo mejor



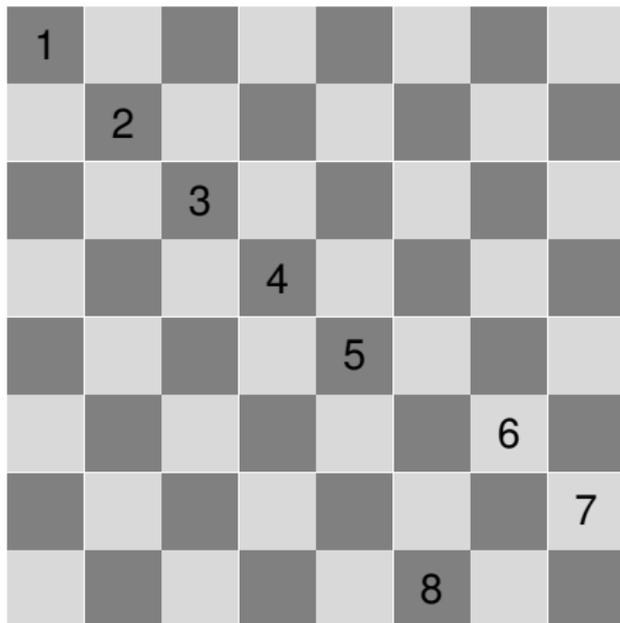
Ocho reinas, un algoritmo mejor



Ocho reinas, un algoritmo mejor



Ocho reinas, un algoritmo mejor



Ocho reinas, un algoritmo mejor

El algoritmo

```
fun ocho_reinas_3() ret r: nat
```

```
  r := 0
```

```
  or_3([ ], r)
```

```
end
```

```
proc or_3(in sol: list of nat, in/out r: nat)
```

```
  {calcula el número de maneras de extender sol}
```

```
  {hasta ubicar en total 8 reinas sin que se amenacen}
```

```
if |sol| = 8 then
```

```
  if solucion_3(sol) then r := r+1 fi
```

```
else for j := 1 to 8 do
```

```
  if j ∉ sol then or_3(sol ◁ j, r) fi
```

```
  od
```

```
fi
```

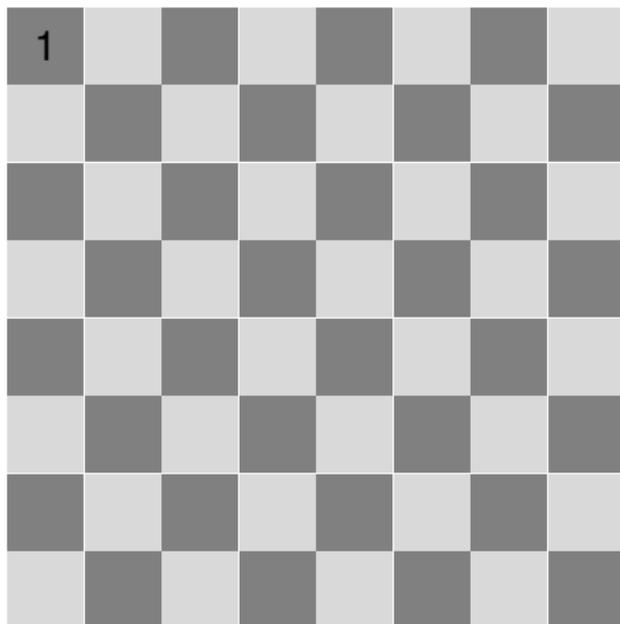
Ocho reinas, un algoritmo mejor

El grafo implícito

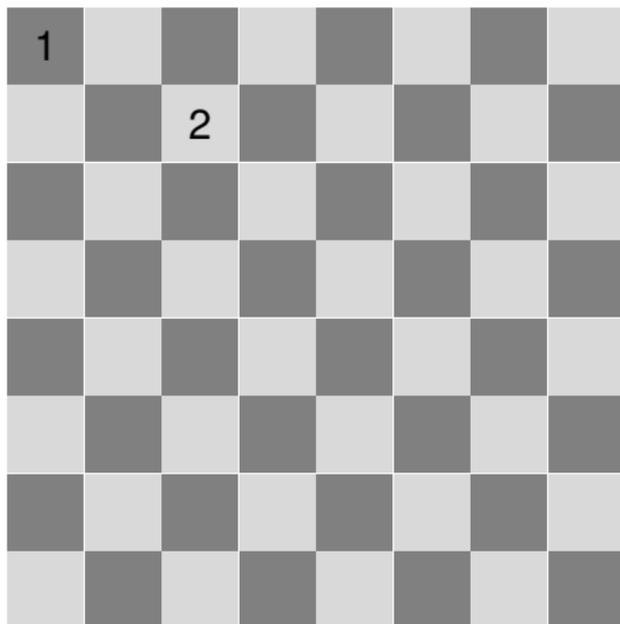
$$V = \{p \in \{1, \dots, 8\}^* \mid |p| \leq 8 \wedge p \text{ sin repeticiones}\}$$

Y las aristas se definen como antes.

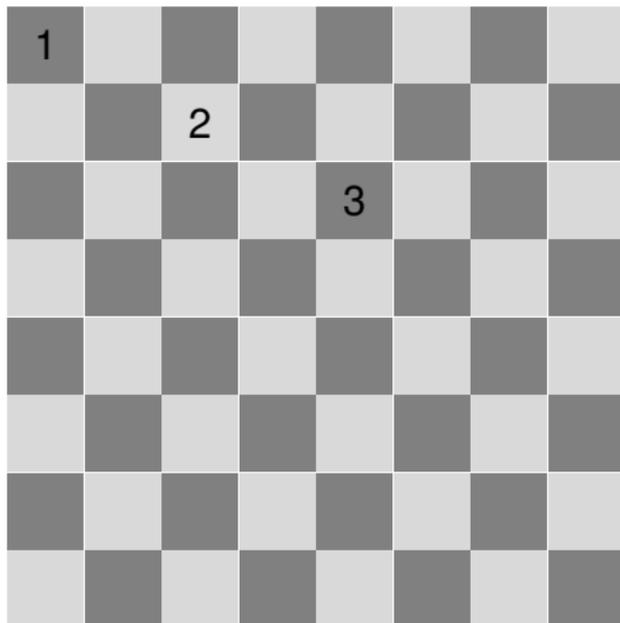
Ocho reinas, un algoritmo optimizado



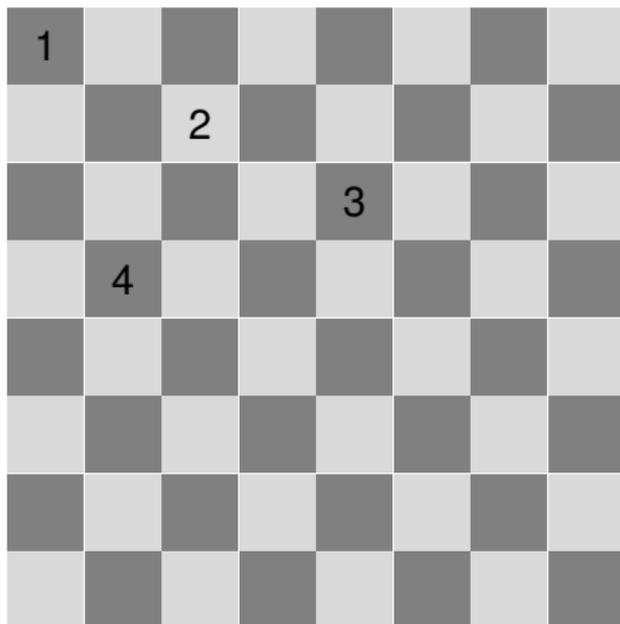
Ocho reinas, un algoritmo optimizado



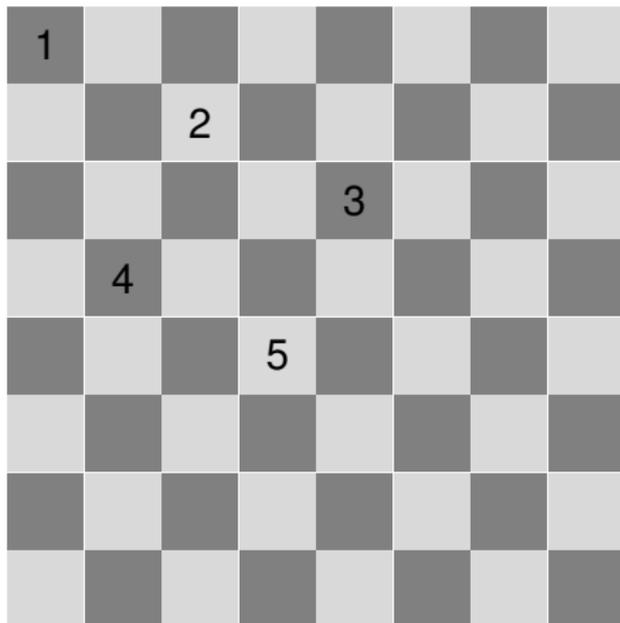
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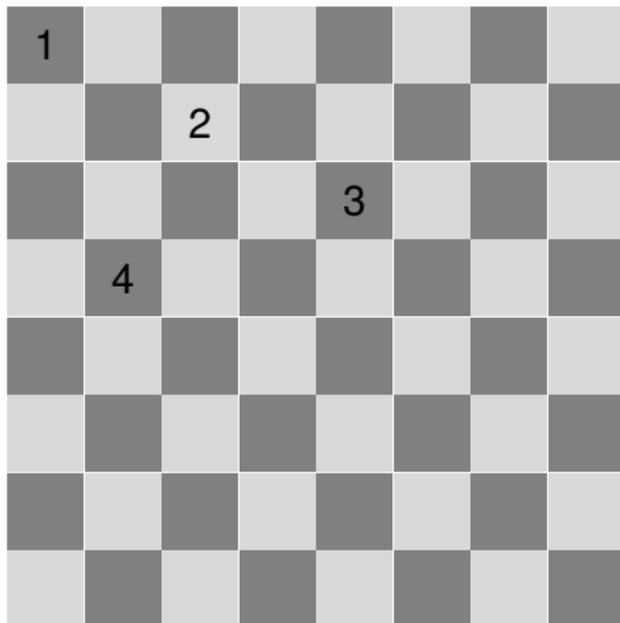
Ocho reinas, un algoritmo optimizado



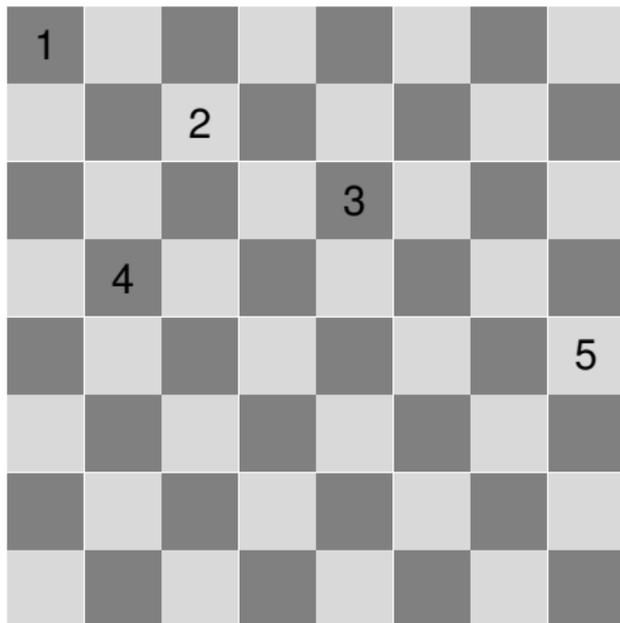
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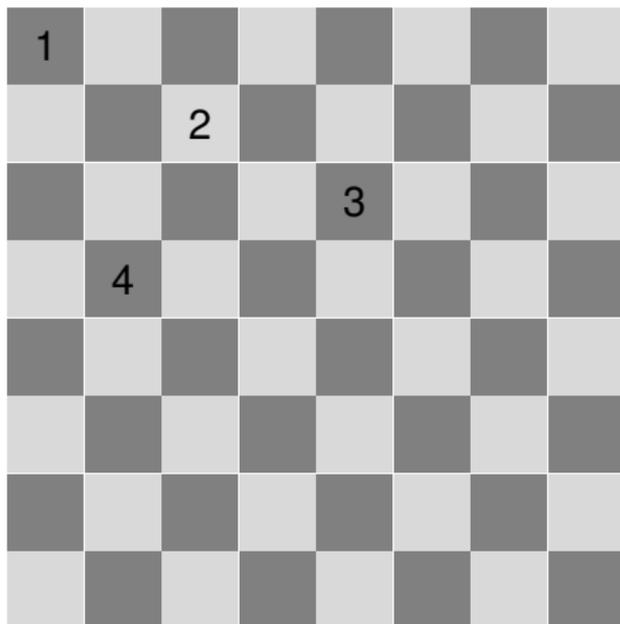
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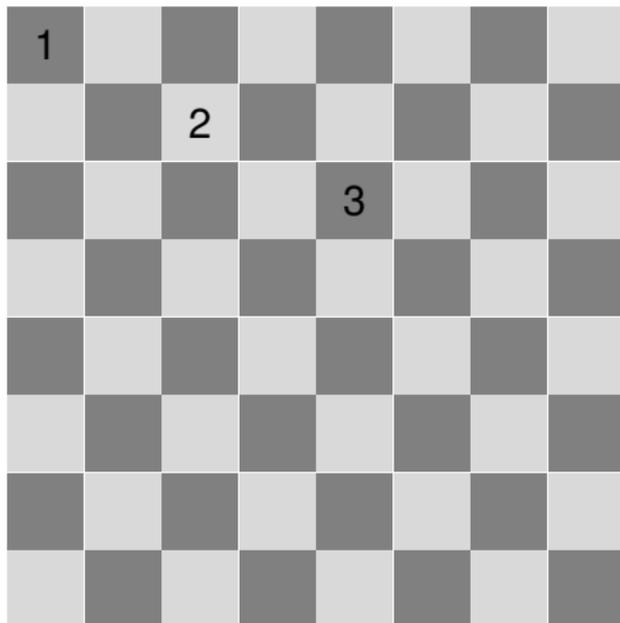
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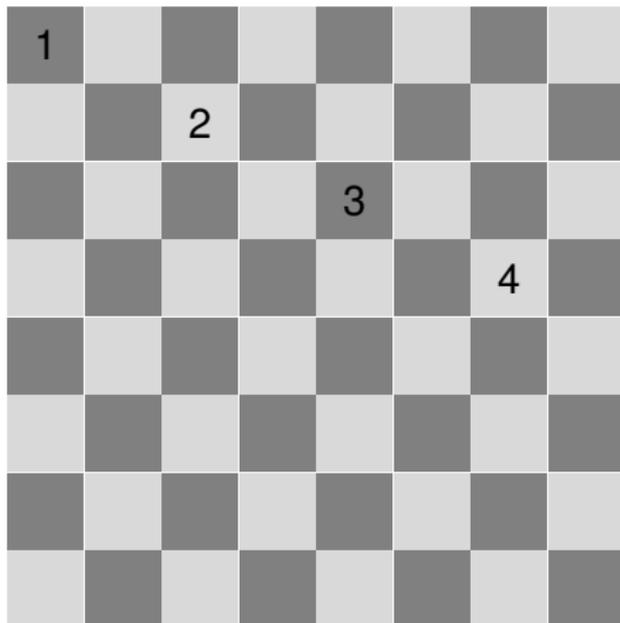
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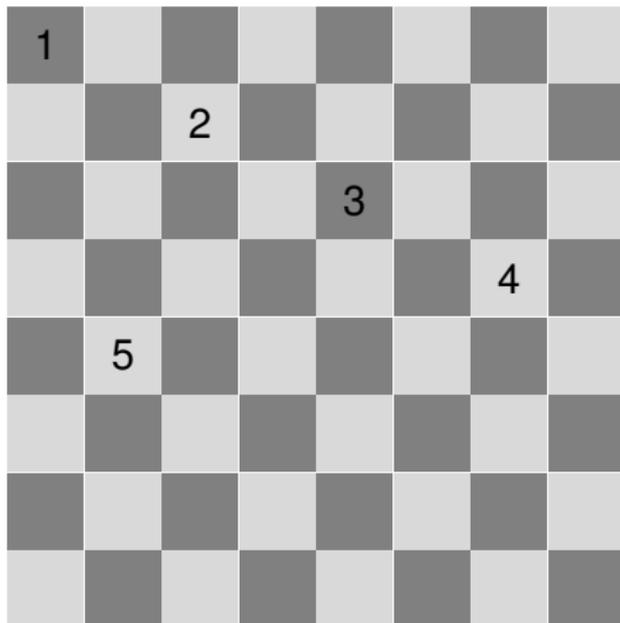
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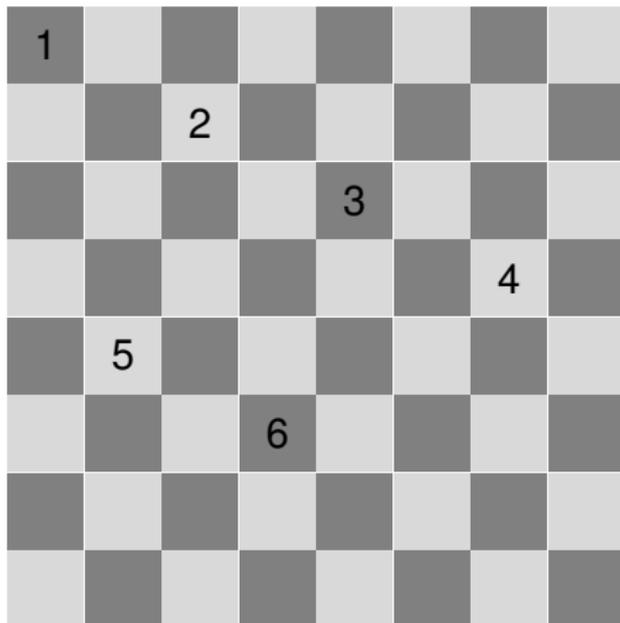
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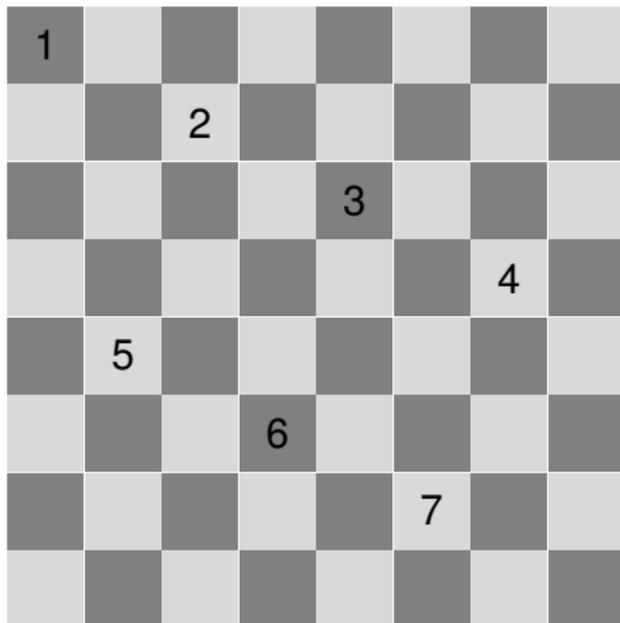
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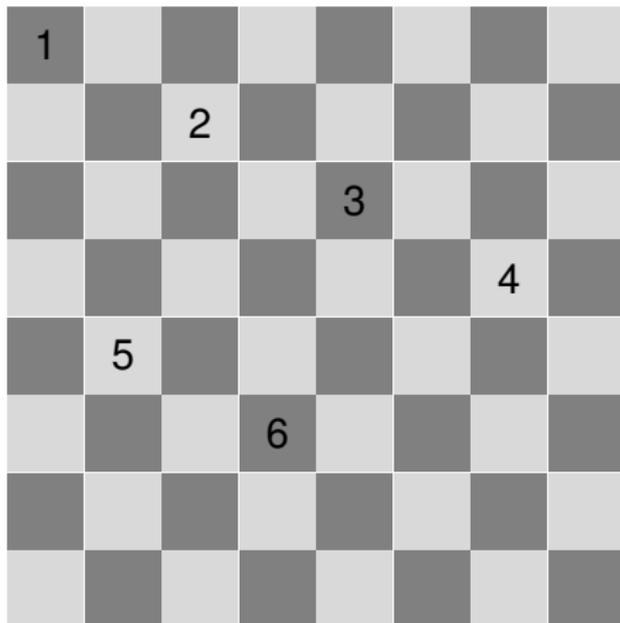
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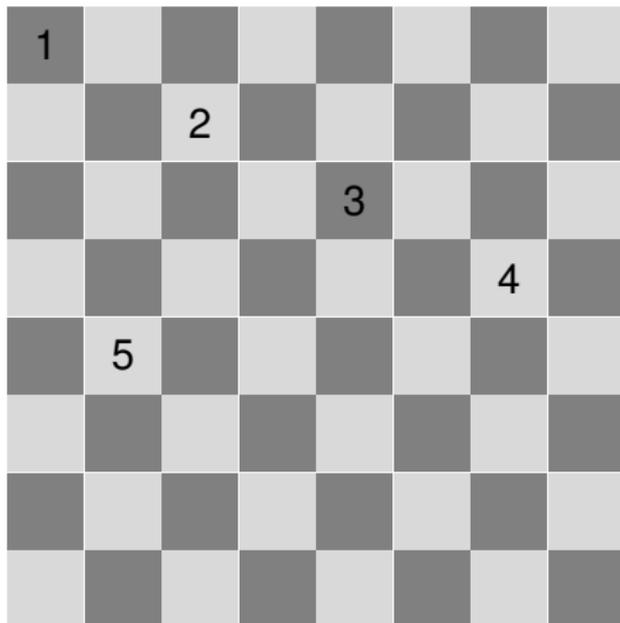
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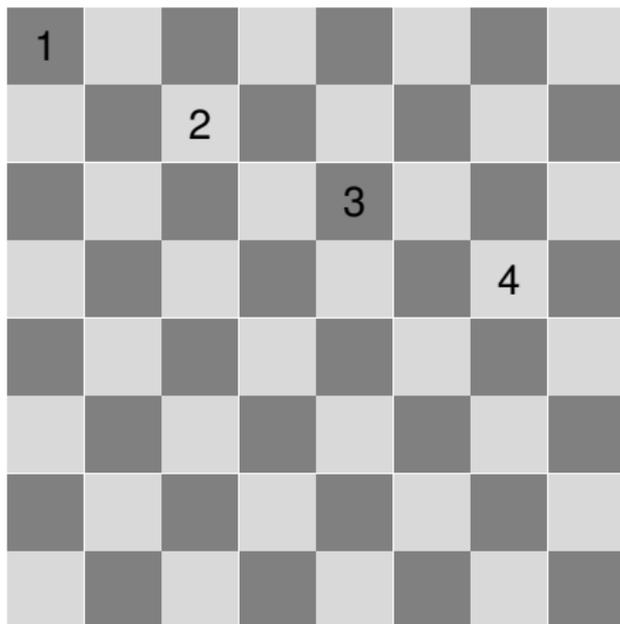
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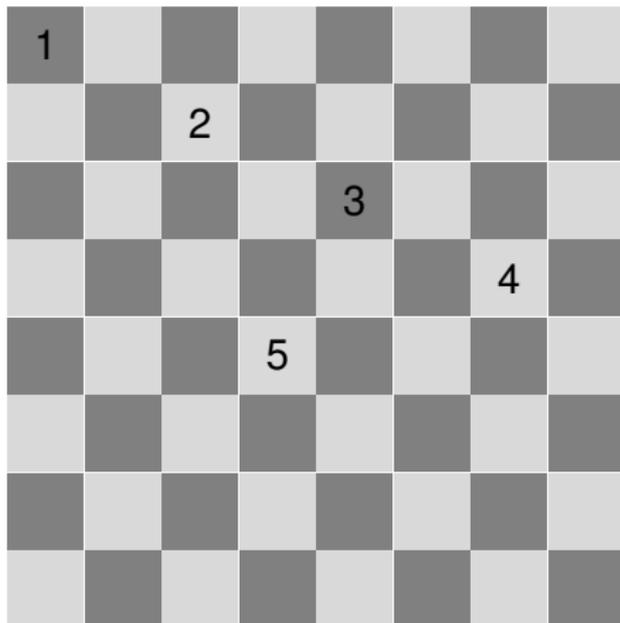
Ocho reinas, un algoritmo optimizado



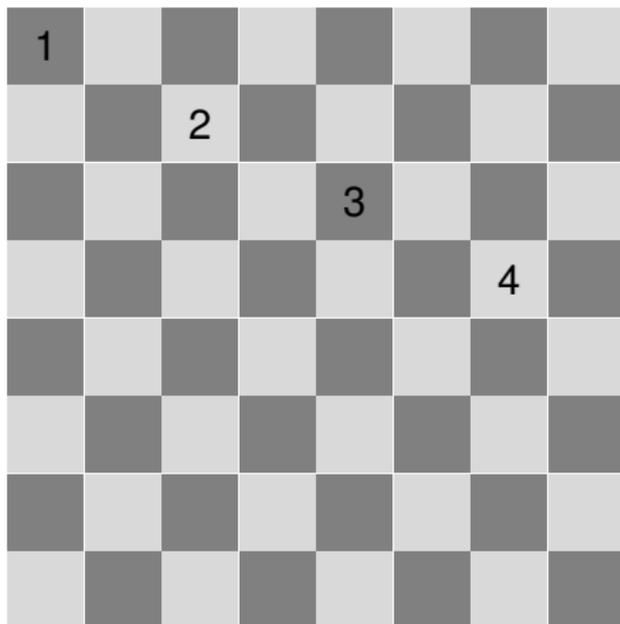
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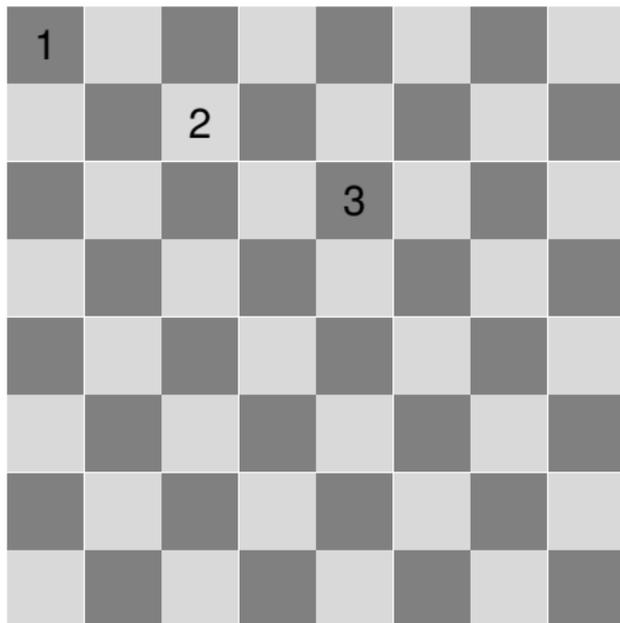
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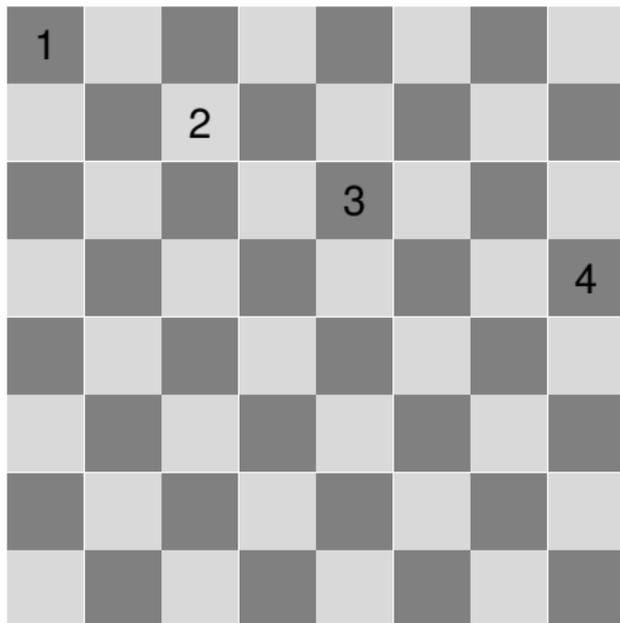
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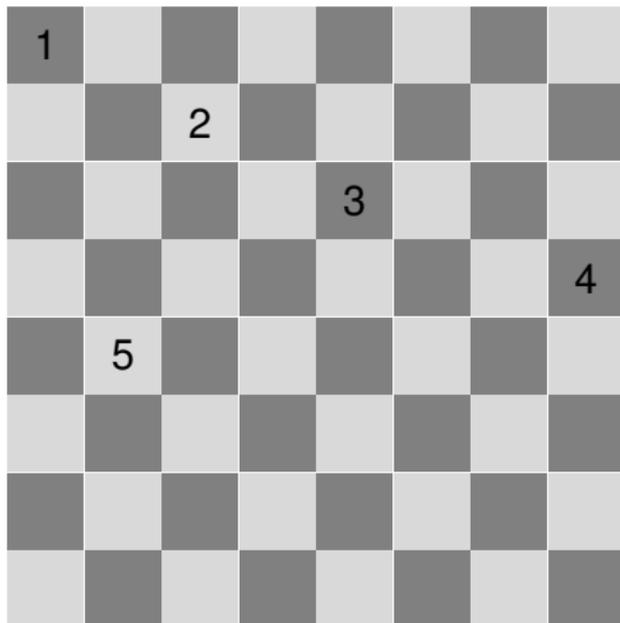
Ocho reinas, un algoritmo optimizado



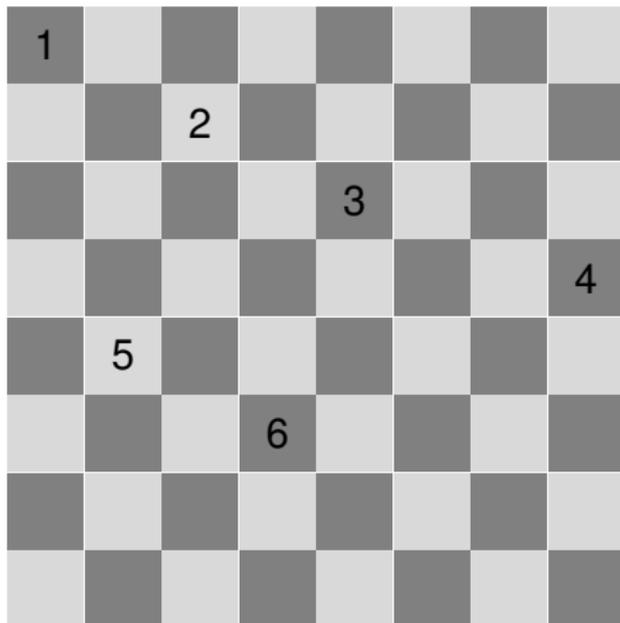
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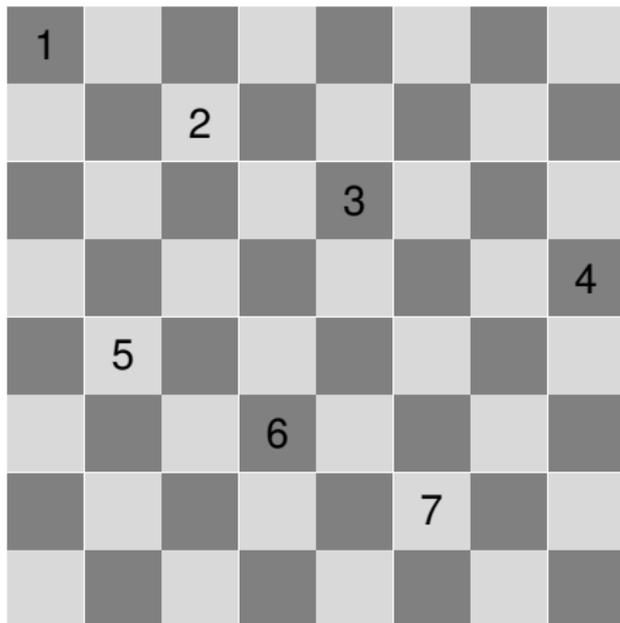
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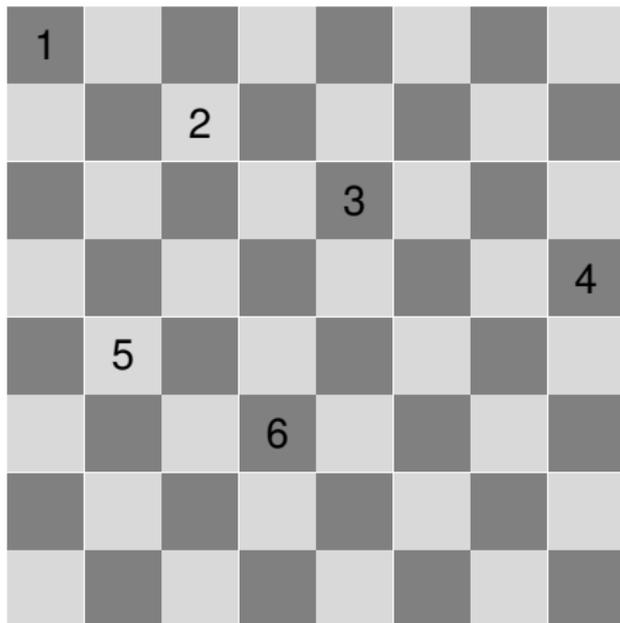
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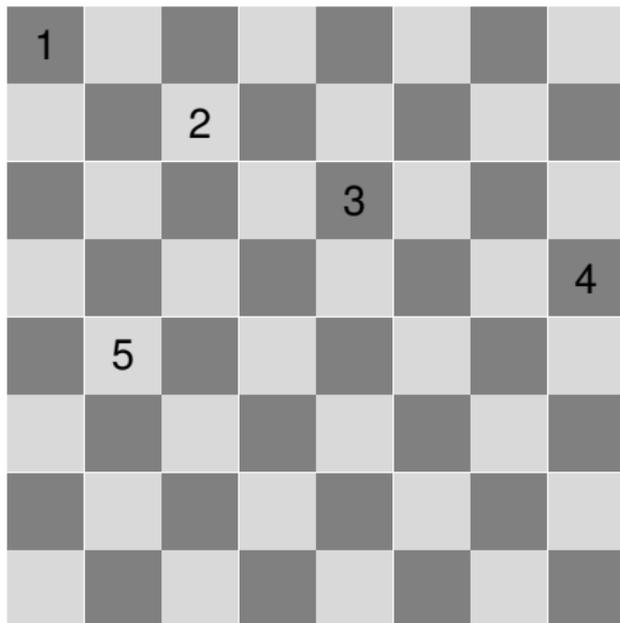
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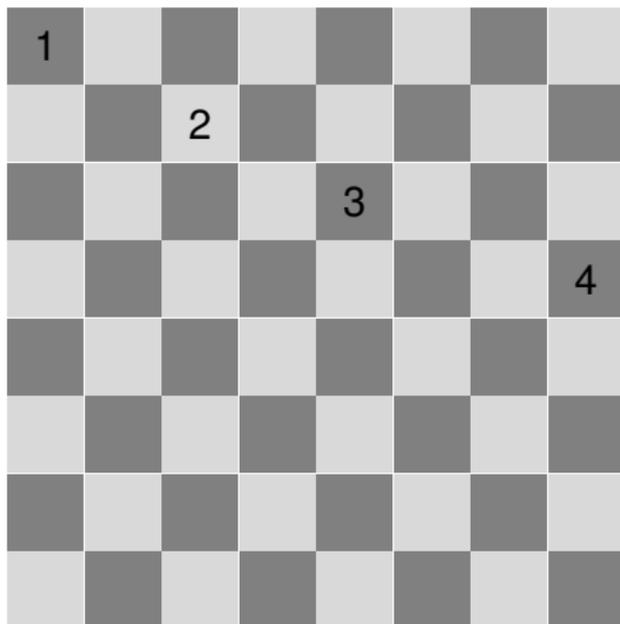
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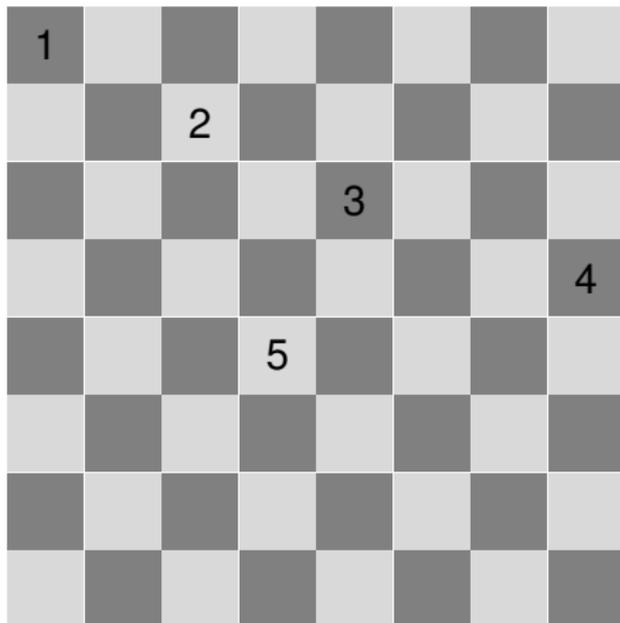
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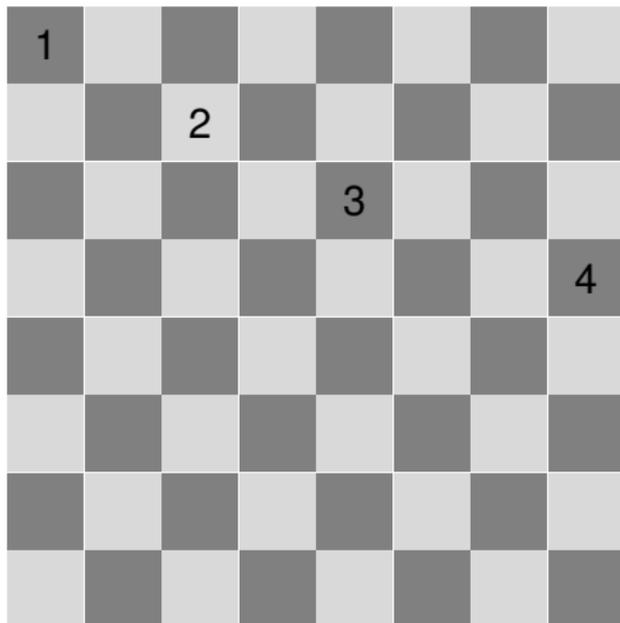
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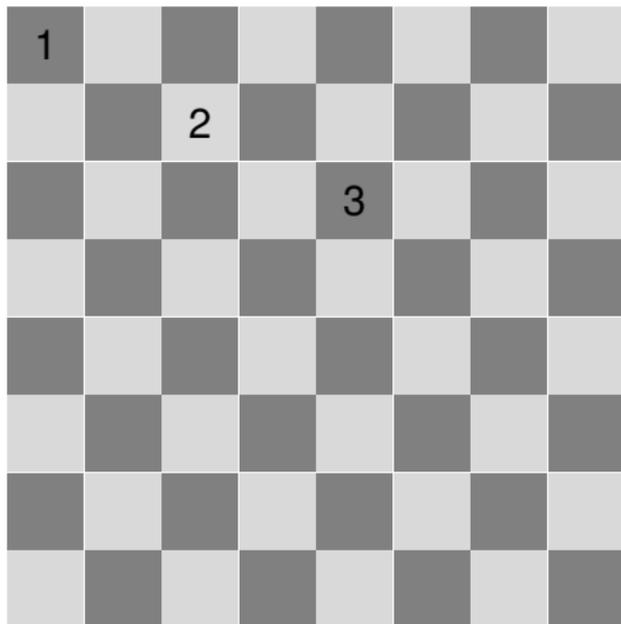
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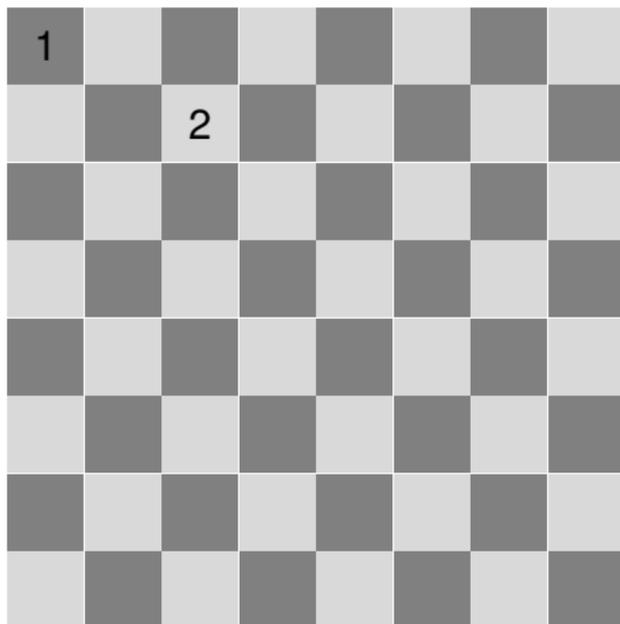
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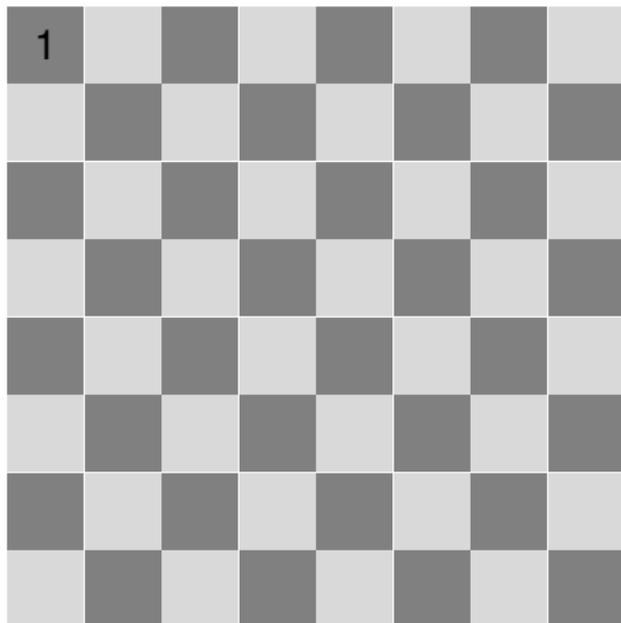
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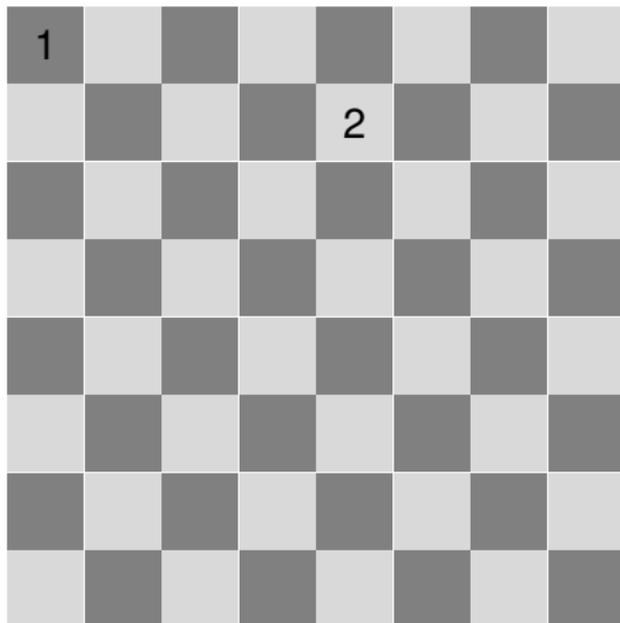
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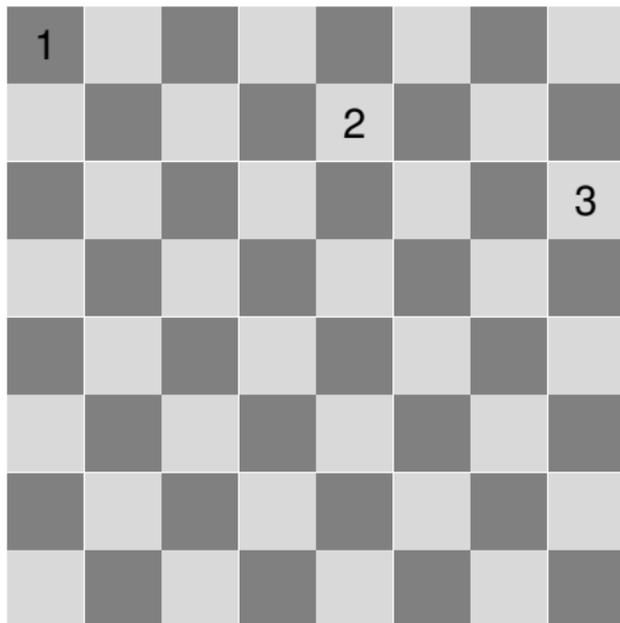
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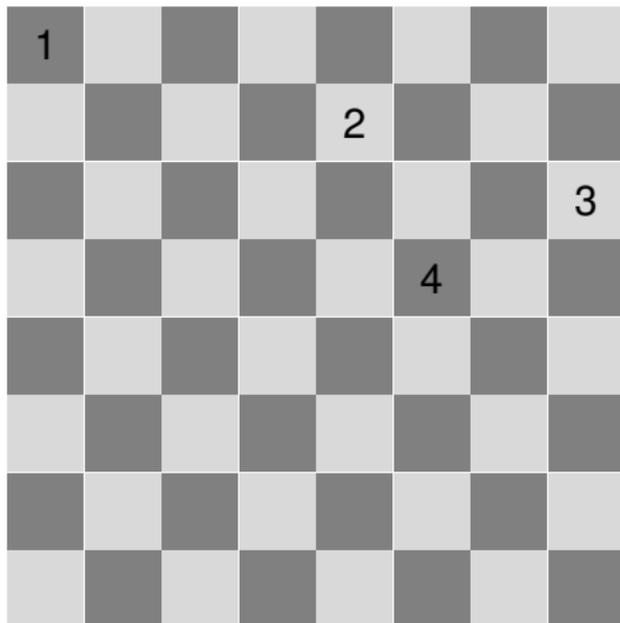
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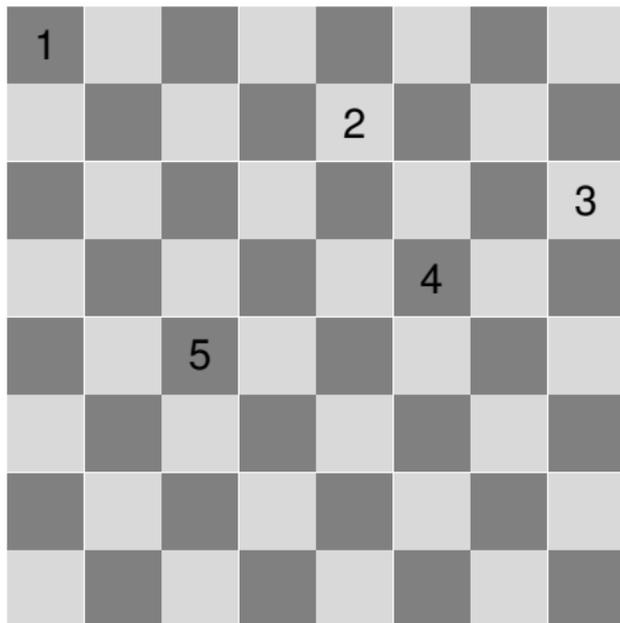
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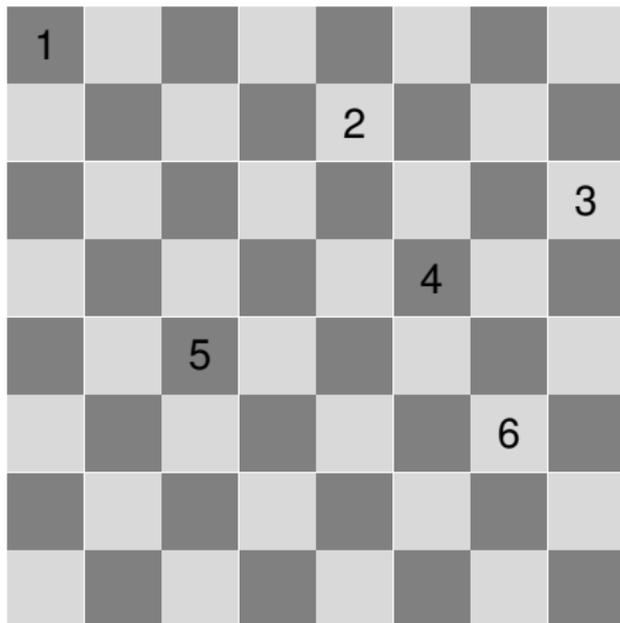
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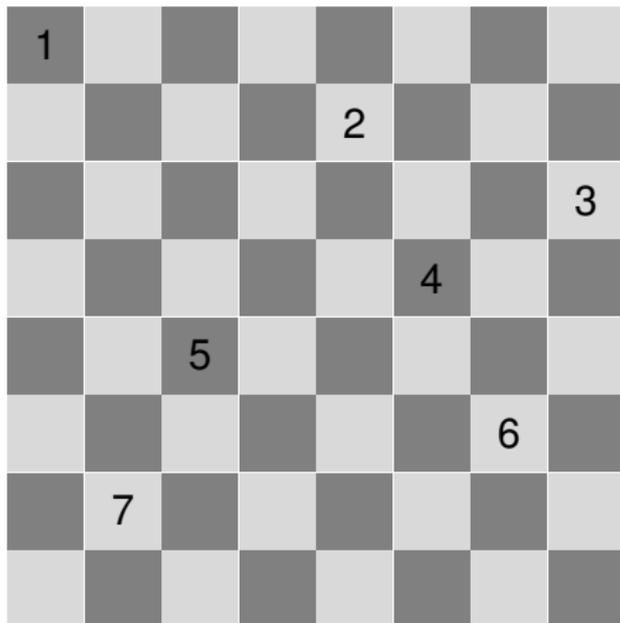
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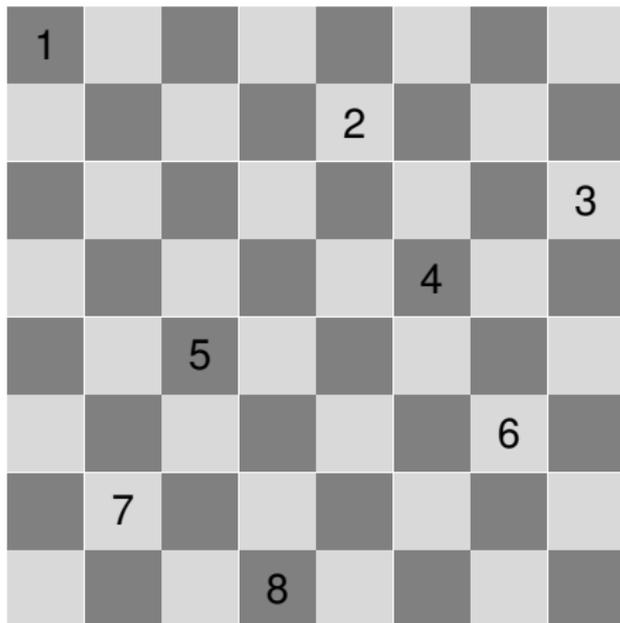
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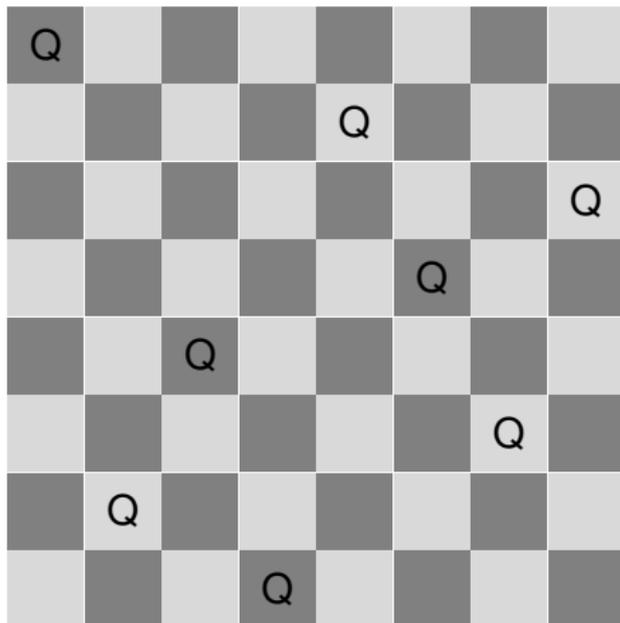
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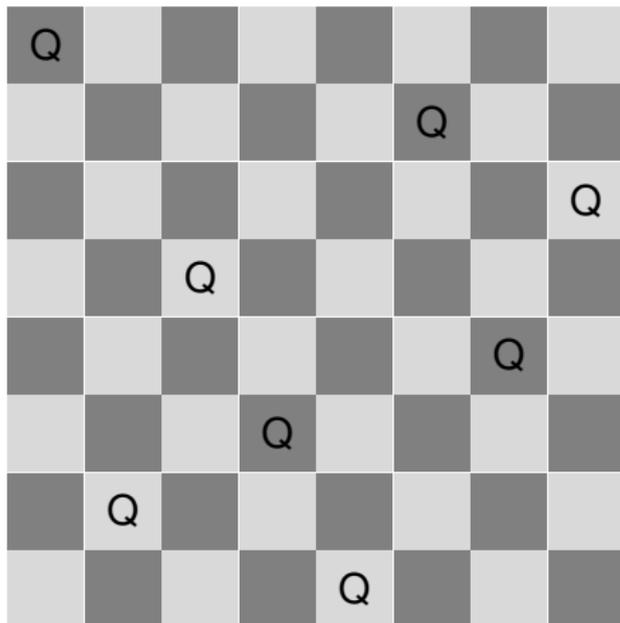
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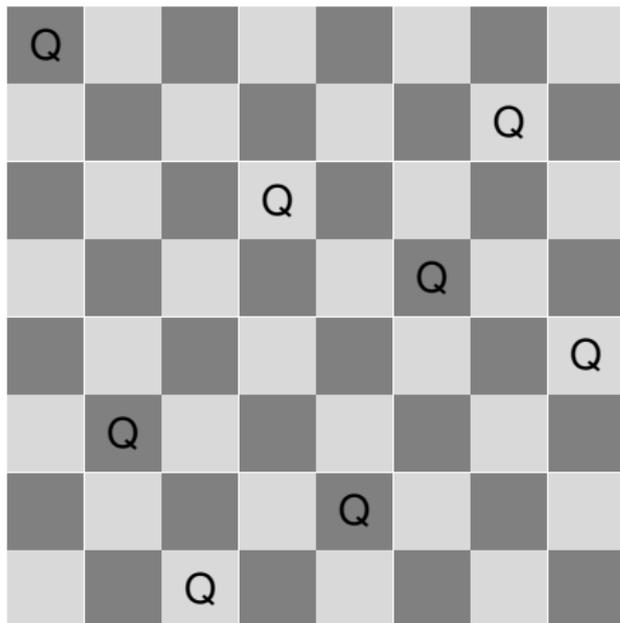
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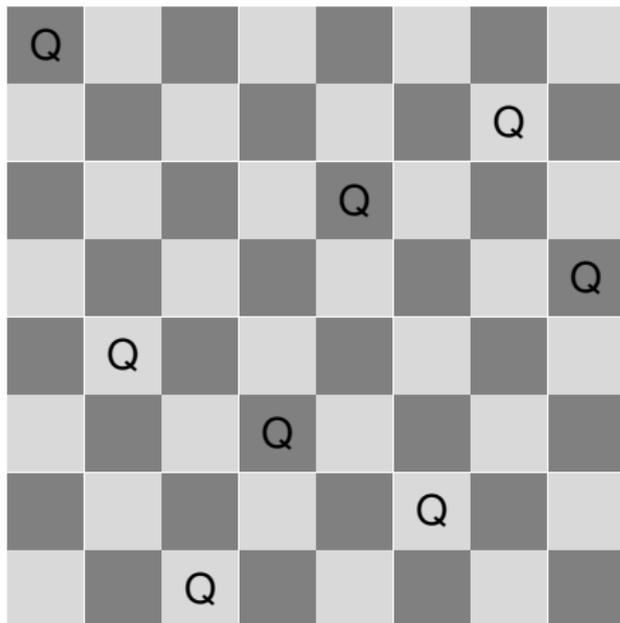
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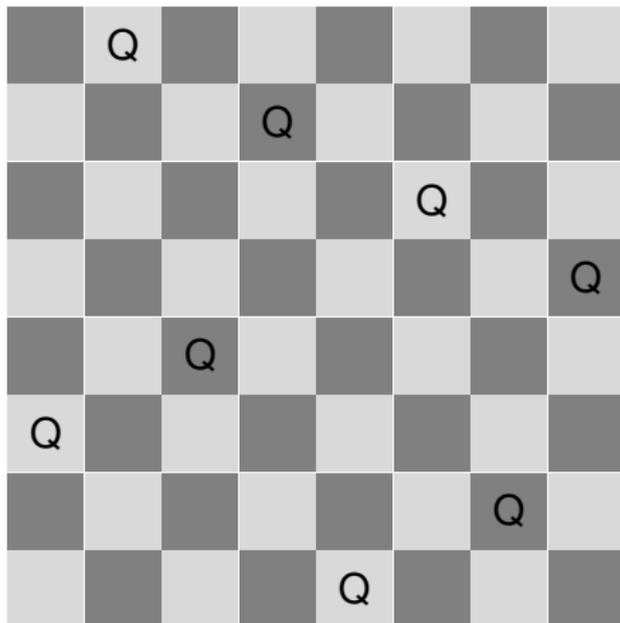
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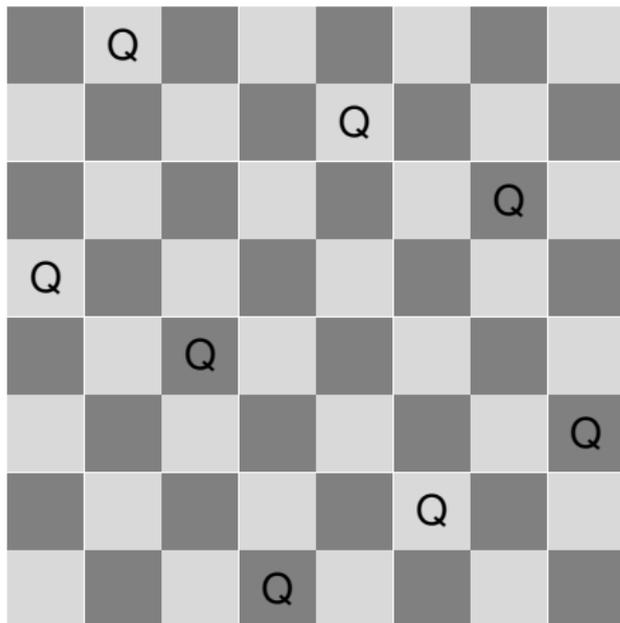
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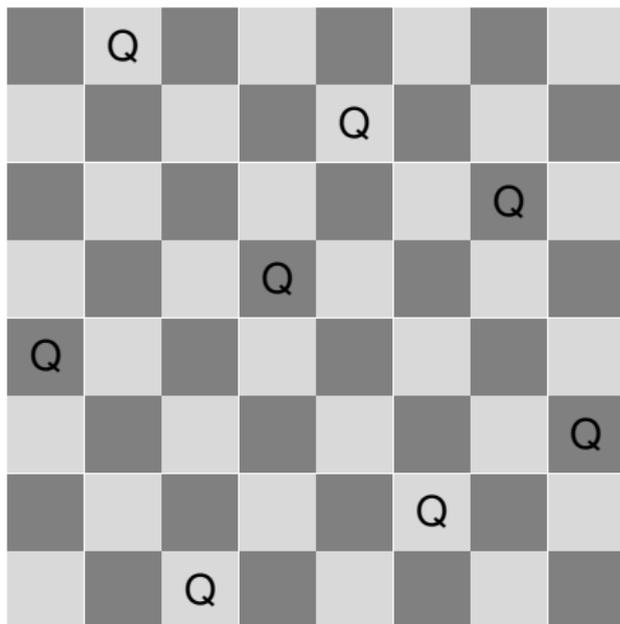
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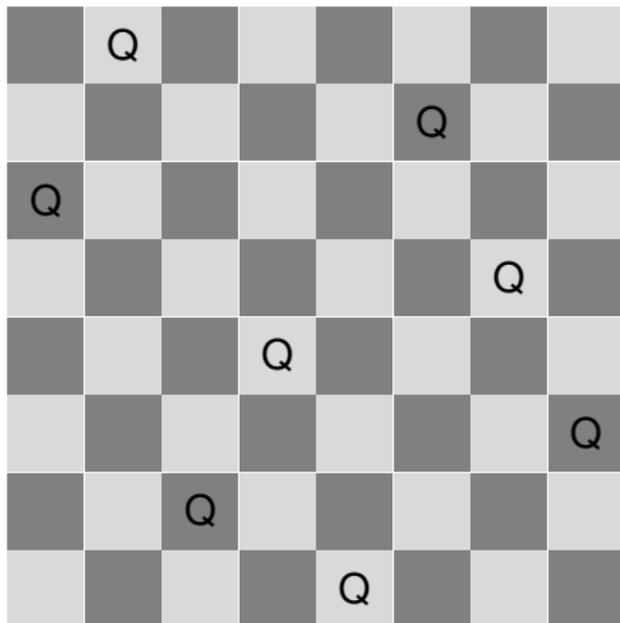
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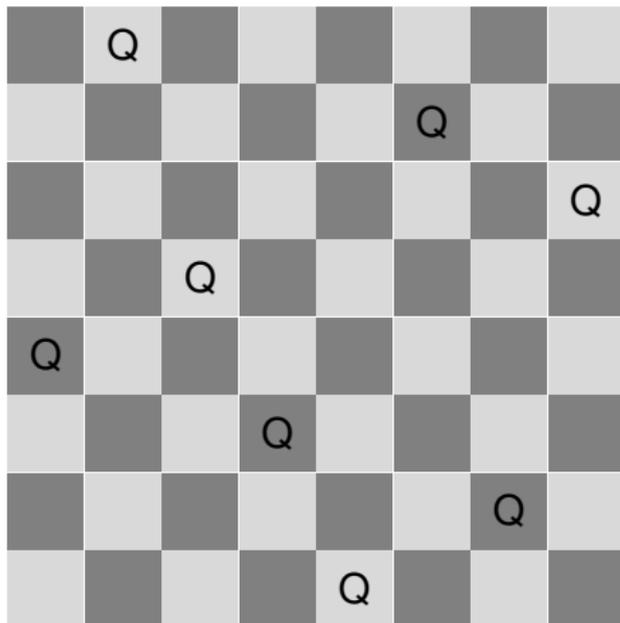
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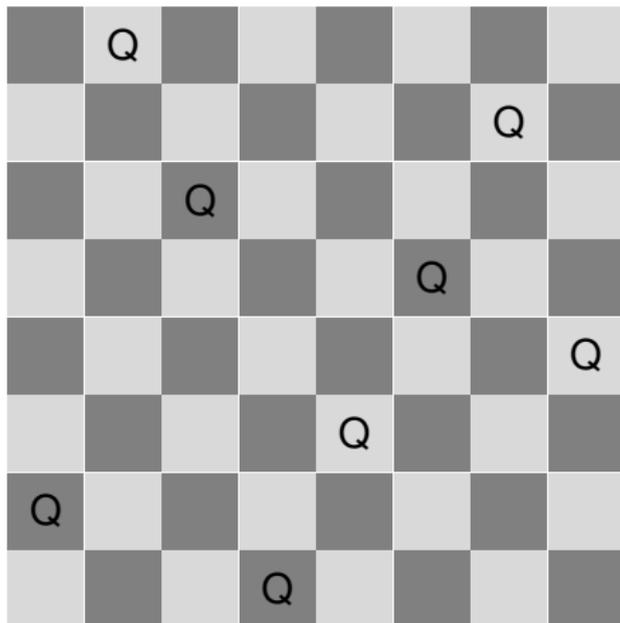
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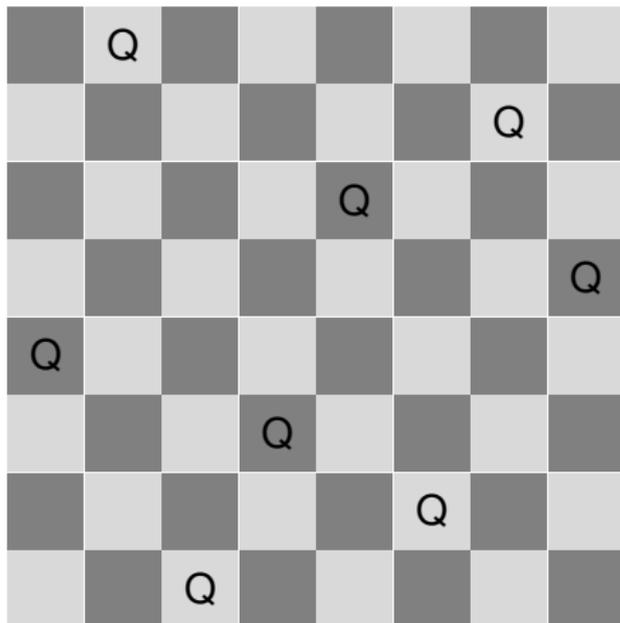
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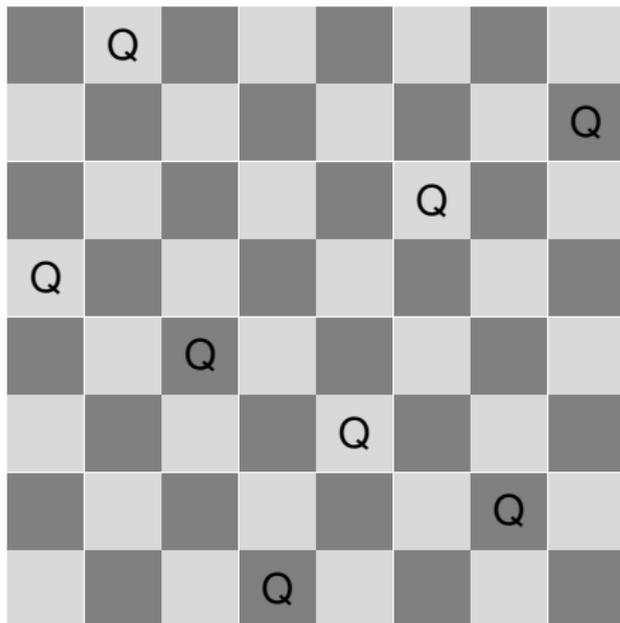
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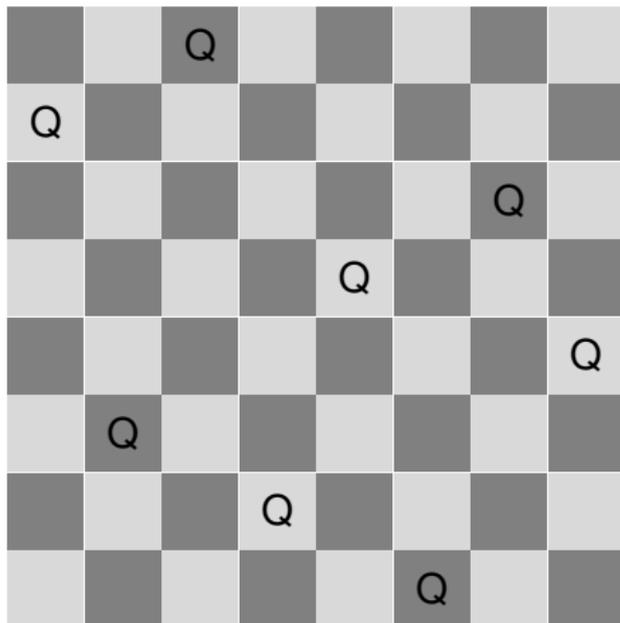
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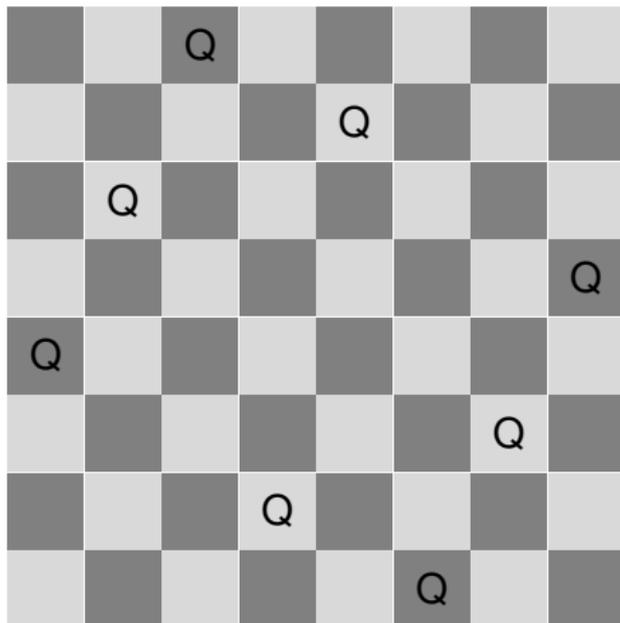
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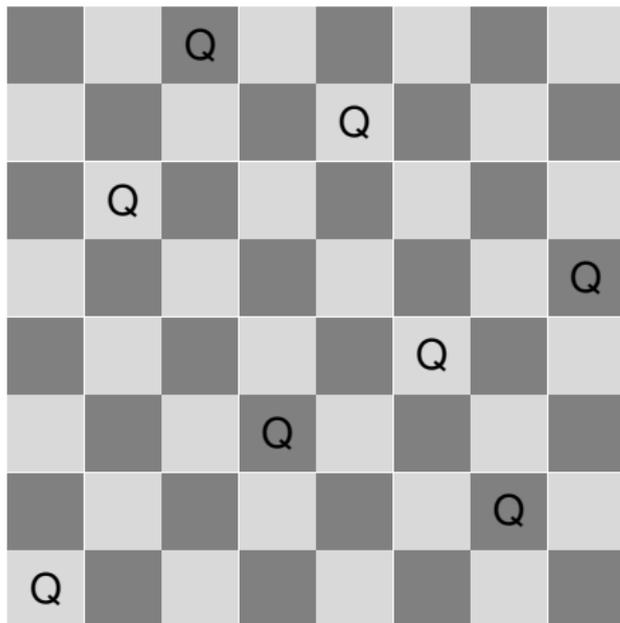
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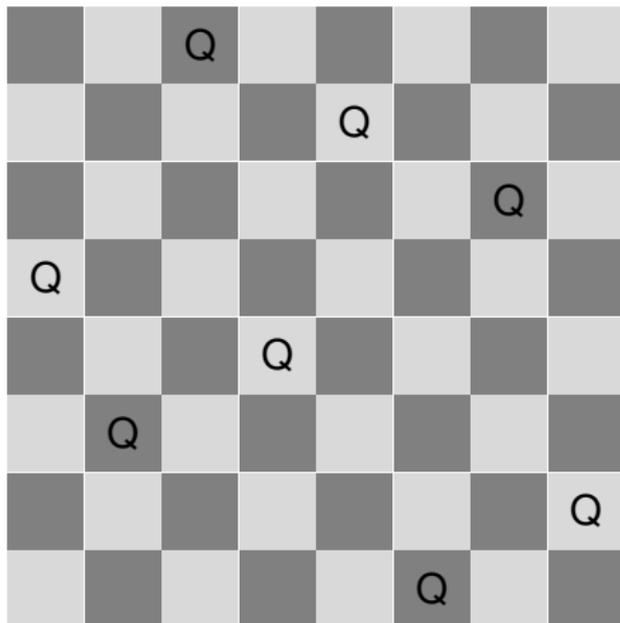
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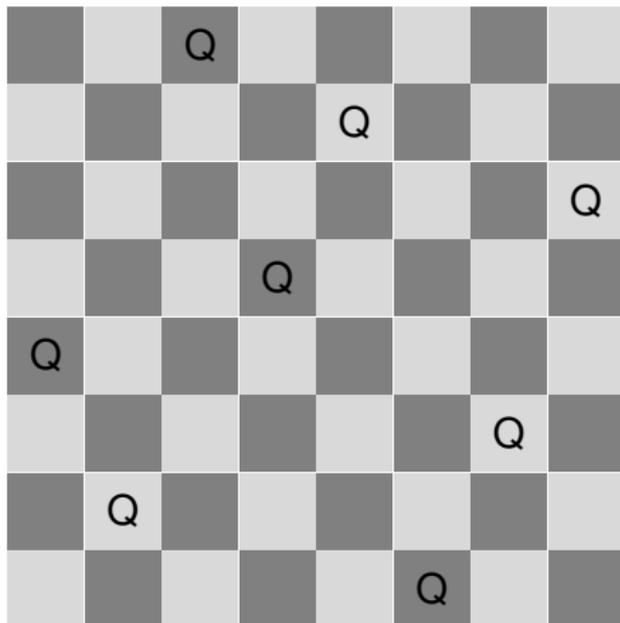
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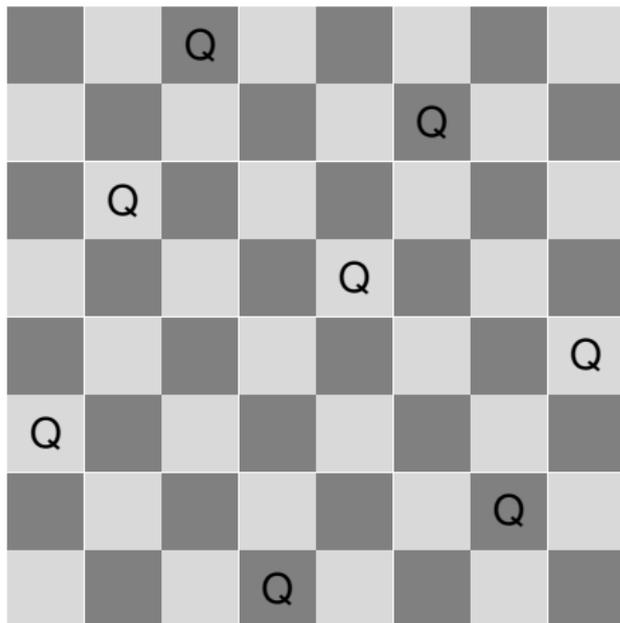
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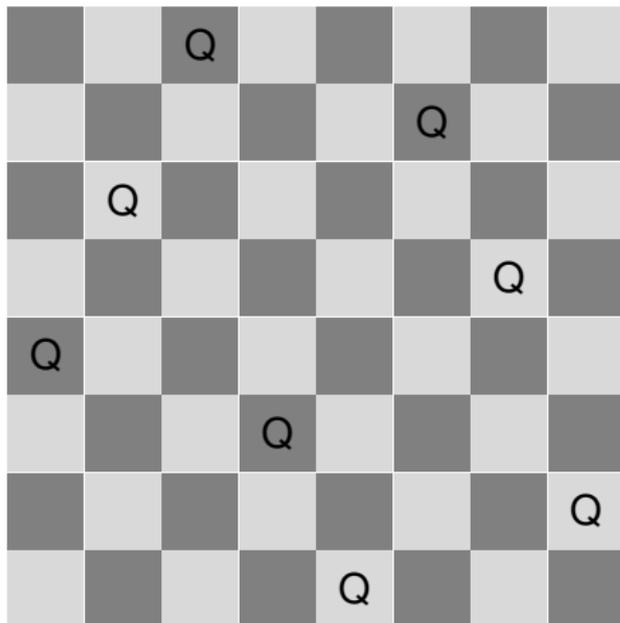
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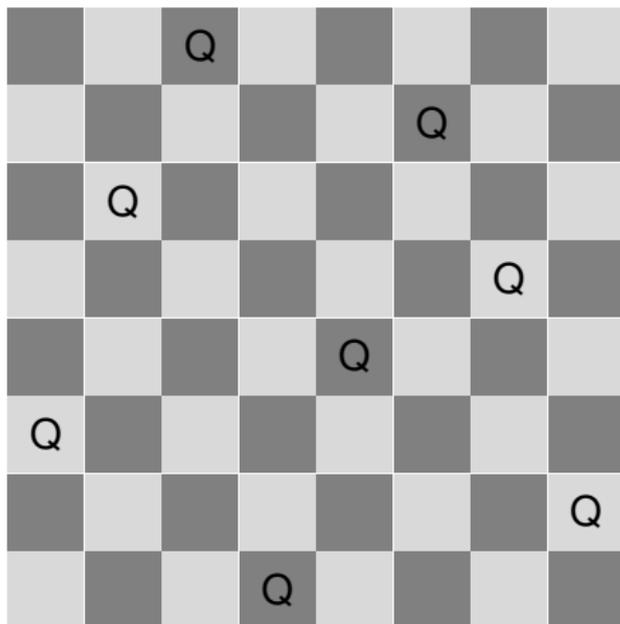
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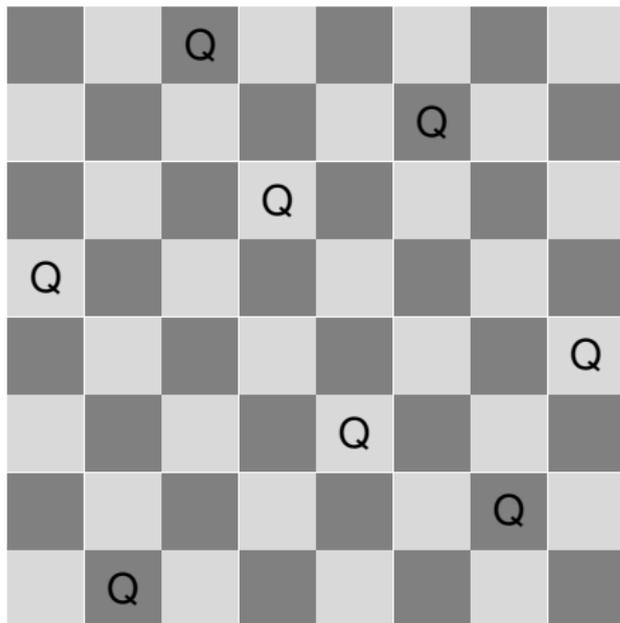
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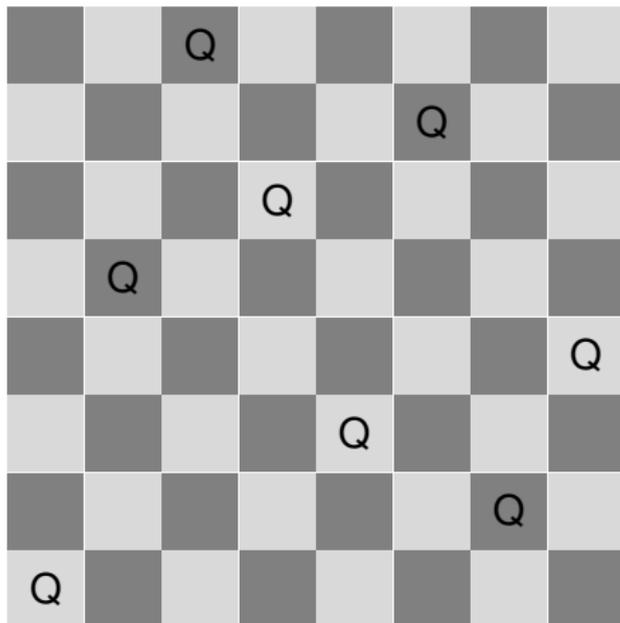
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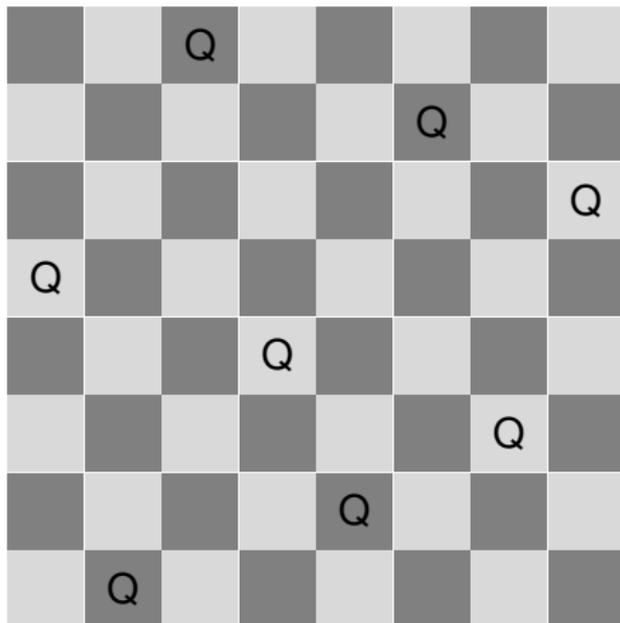
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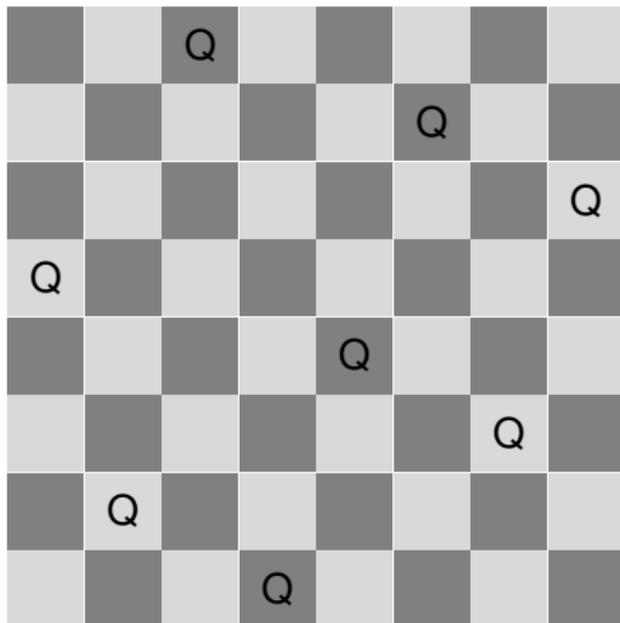
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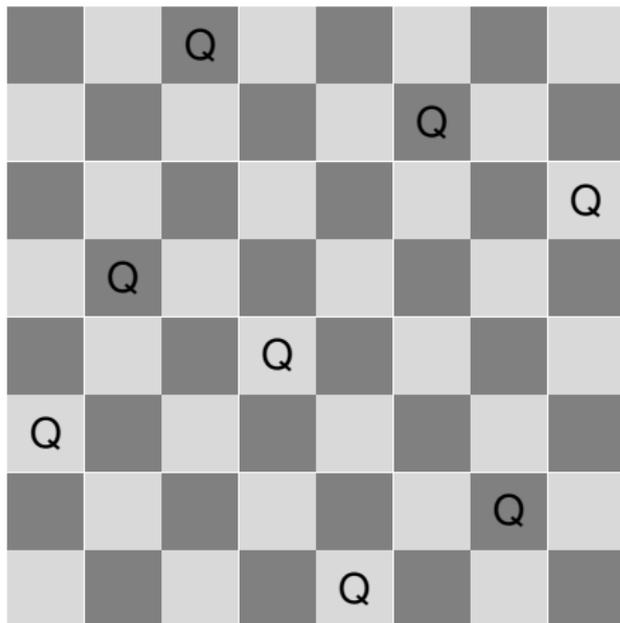
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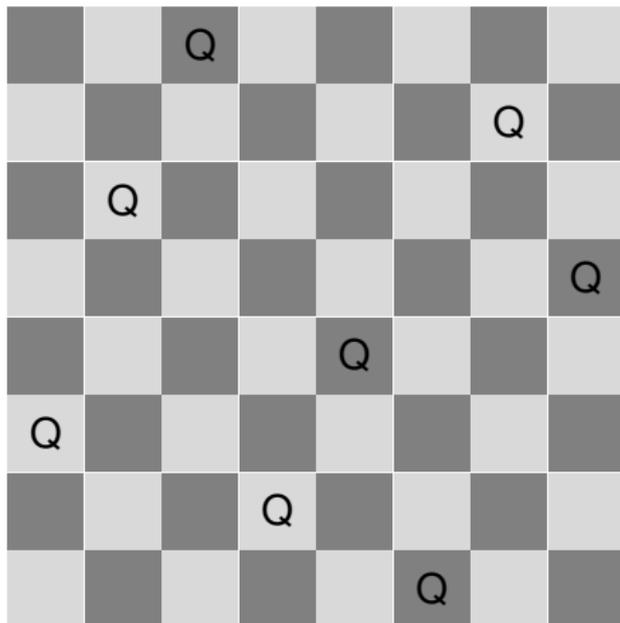
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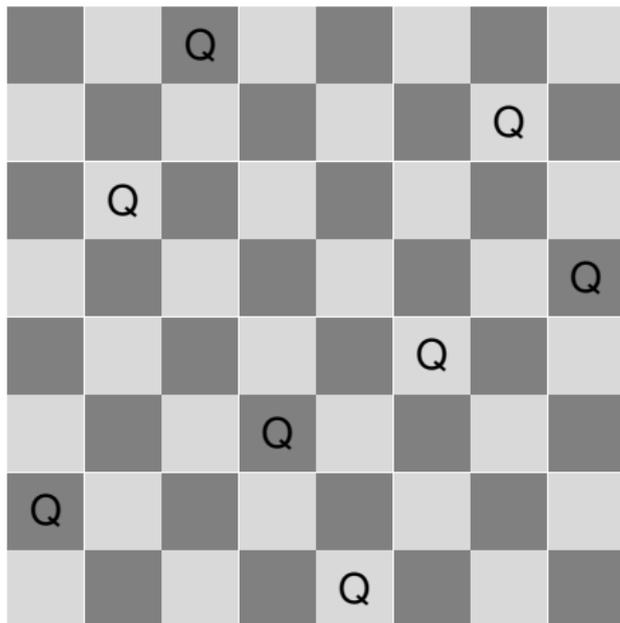
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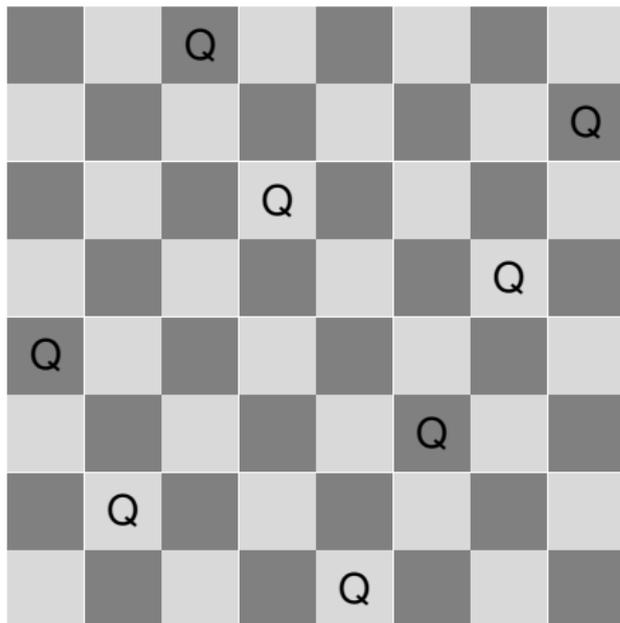
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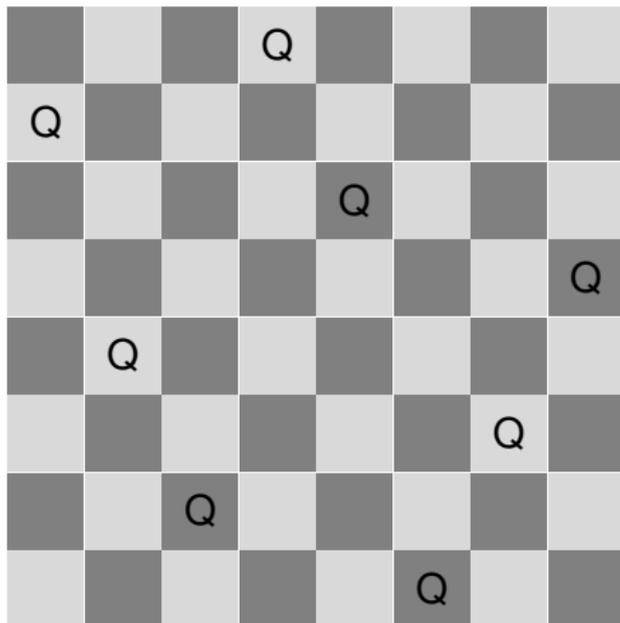
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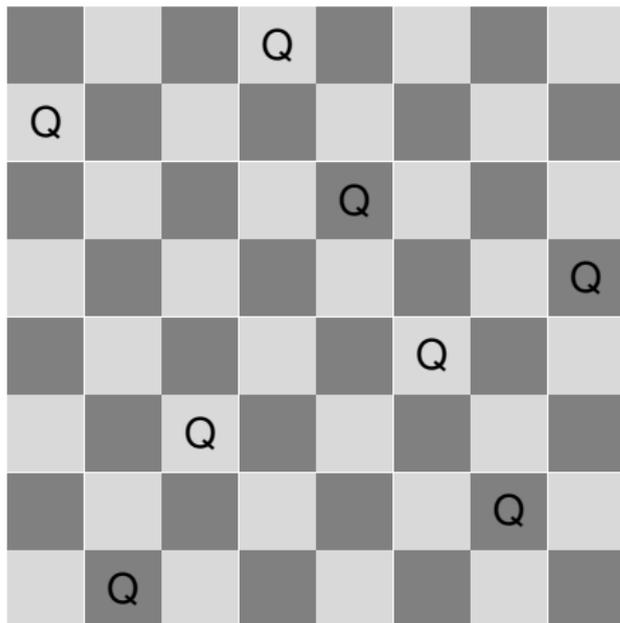
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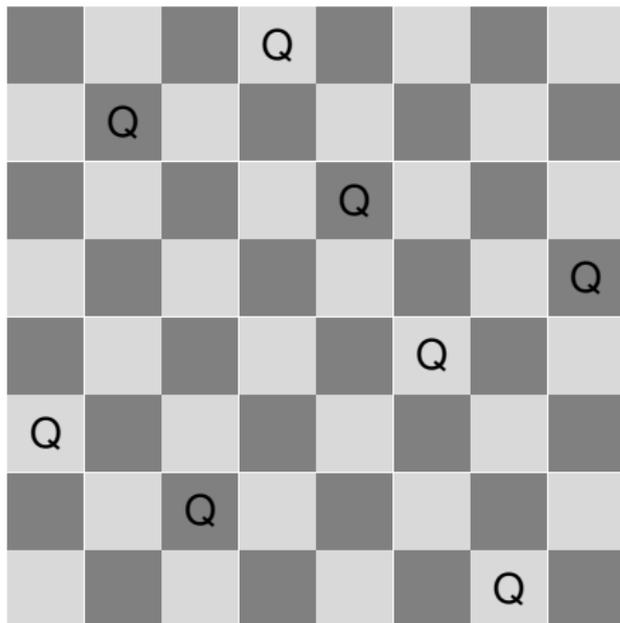
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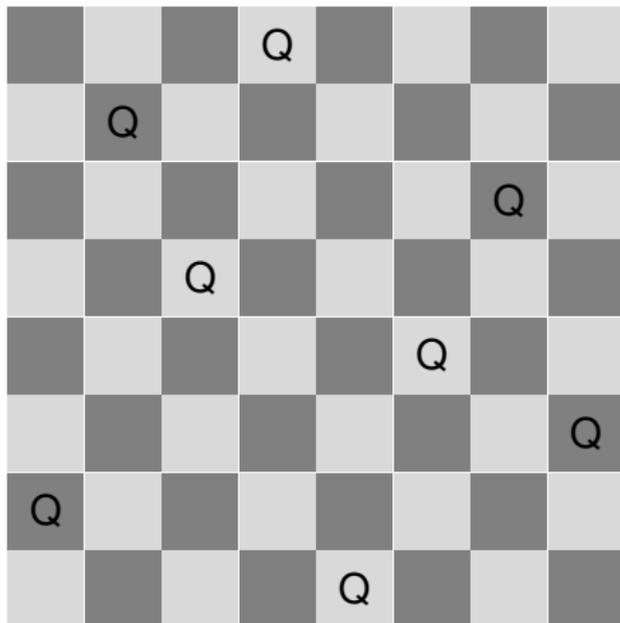
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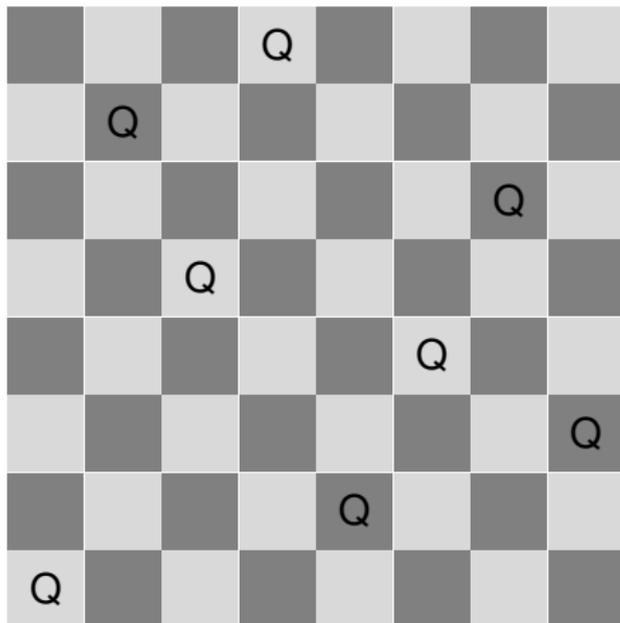
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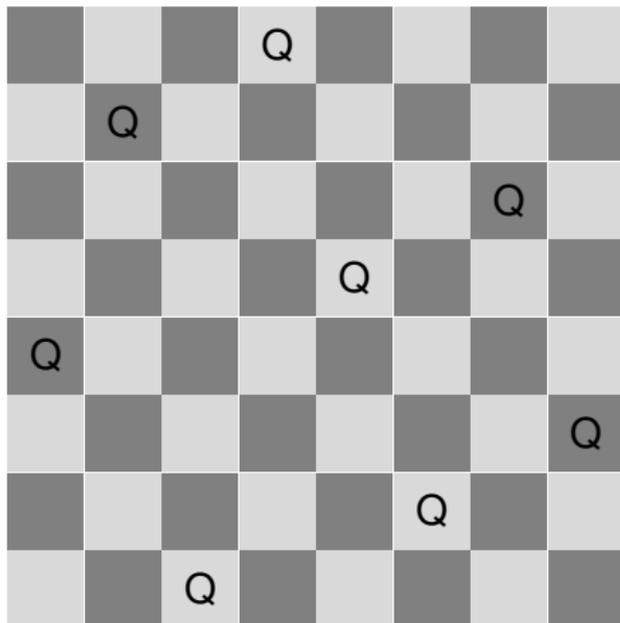
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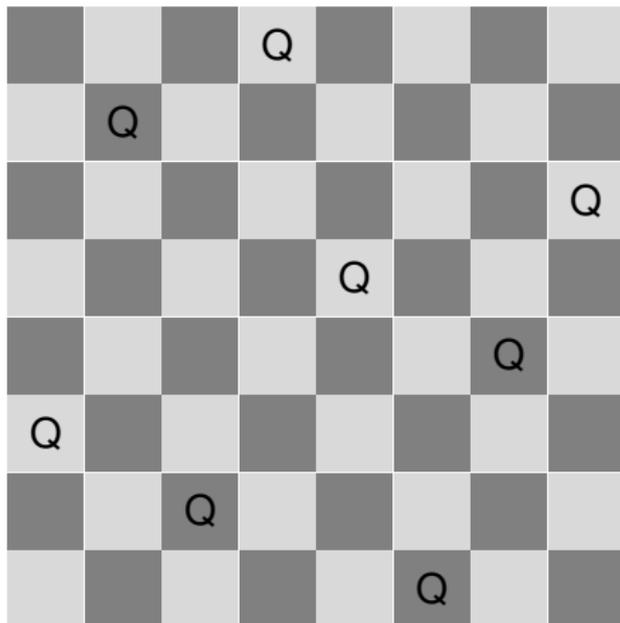
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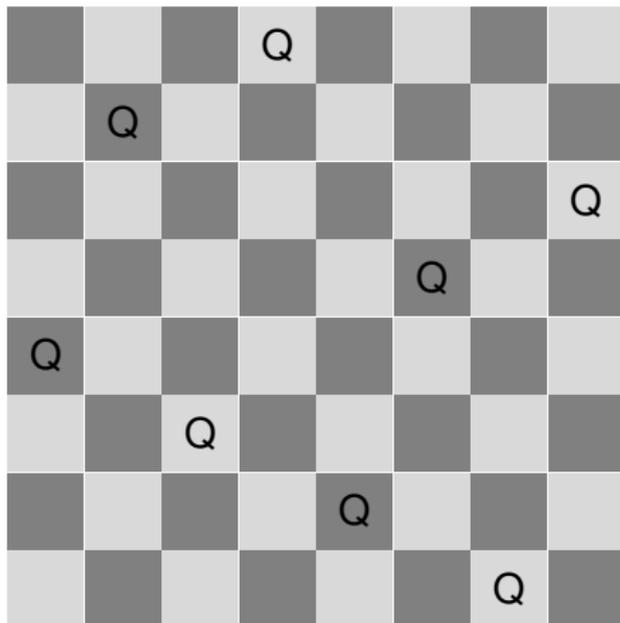
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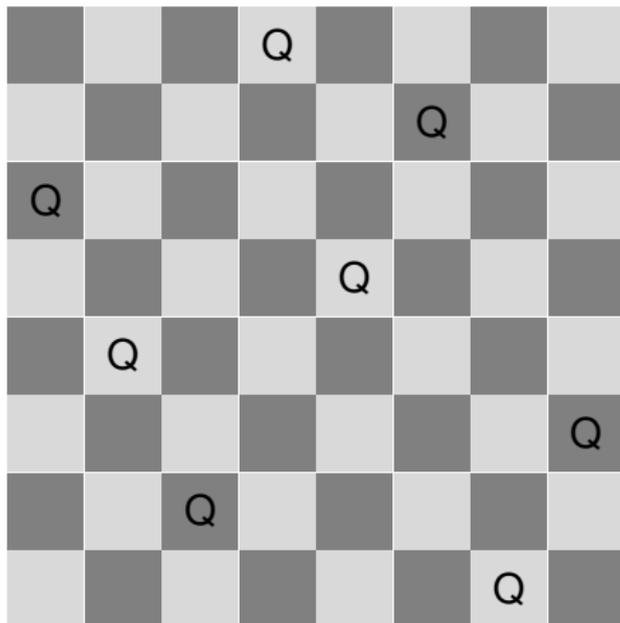
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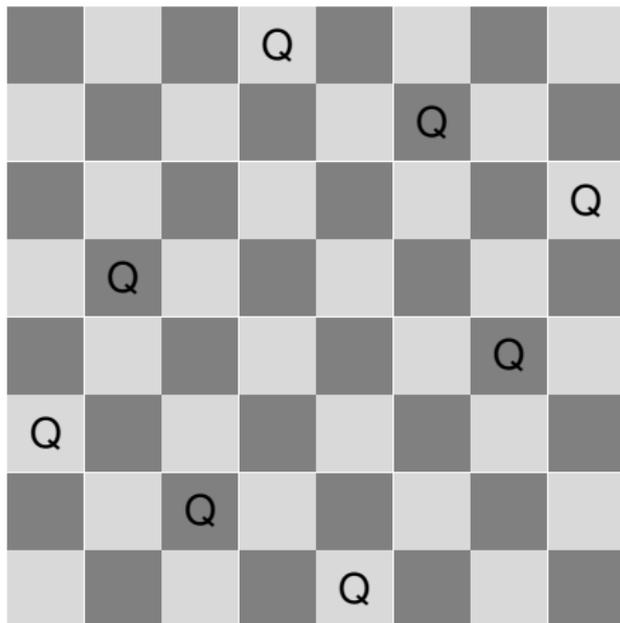
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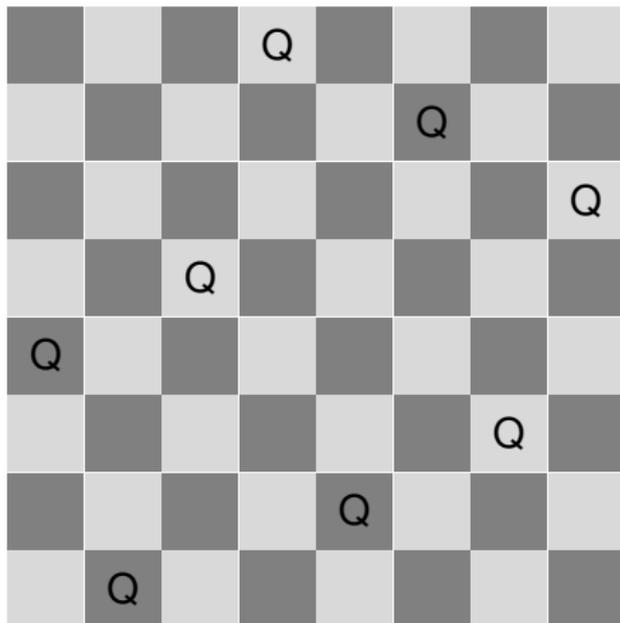
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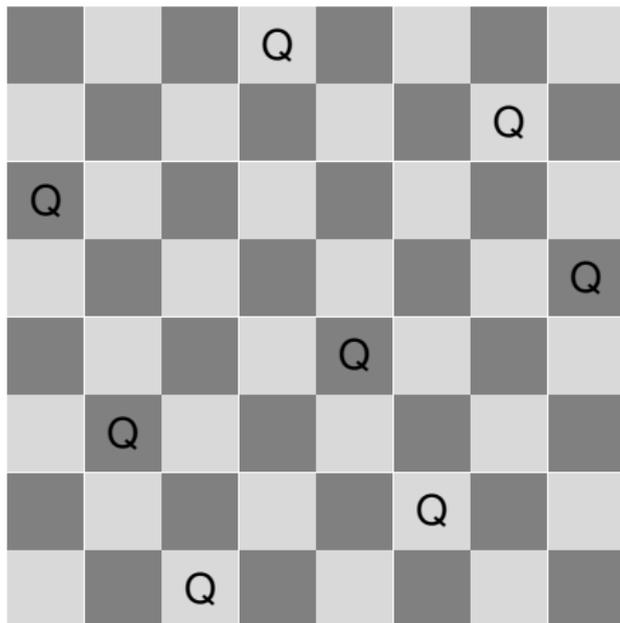
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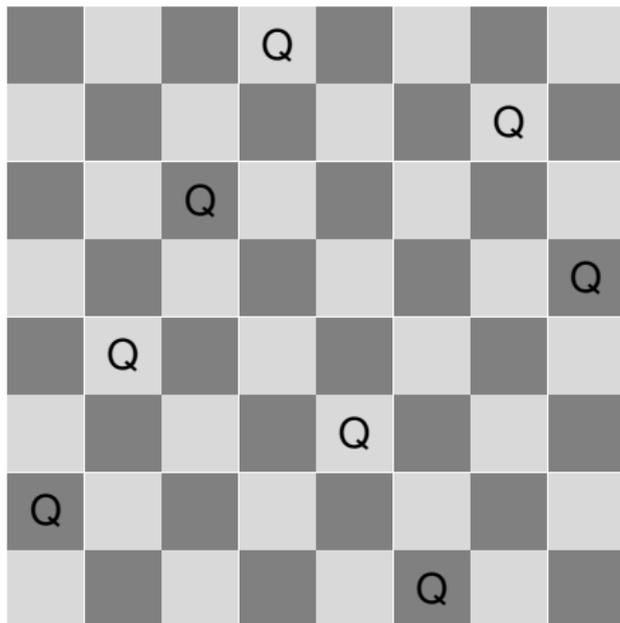
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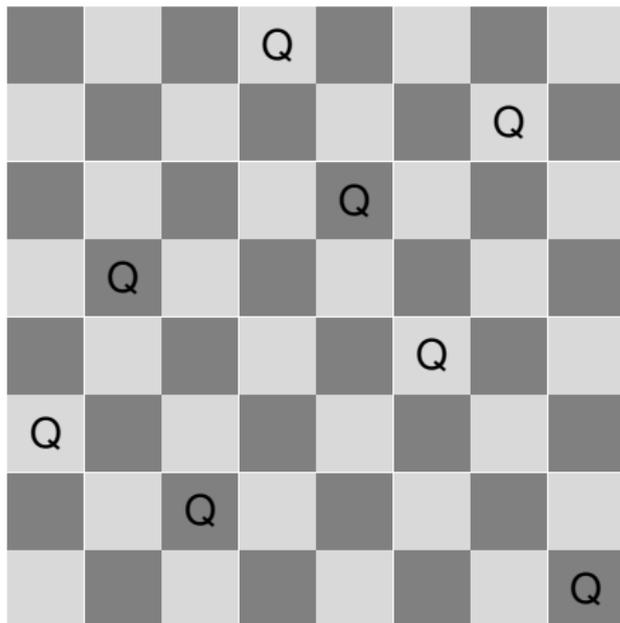
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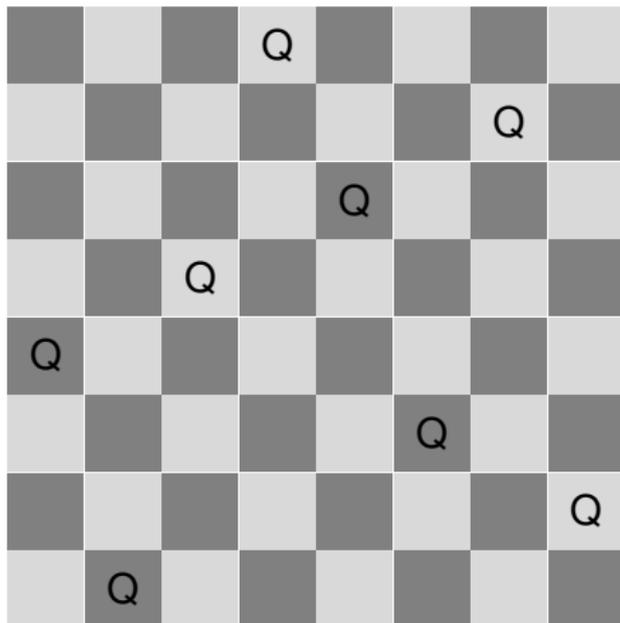
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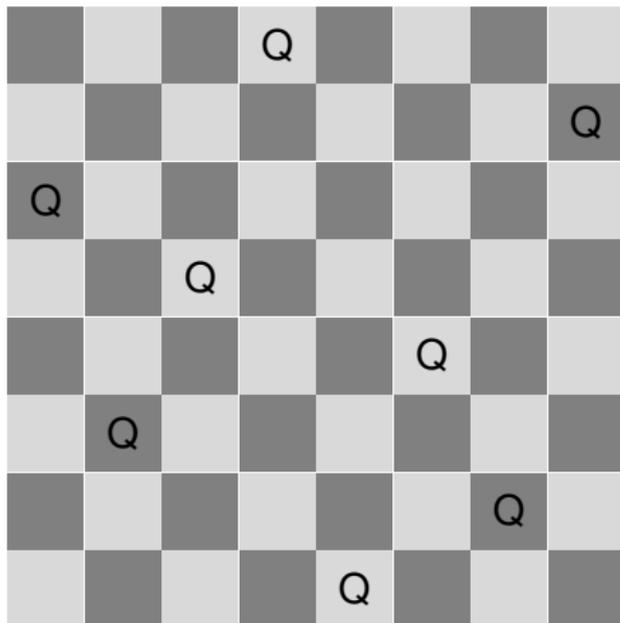
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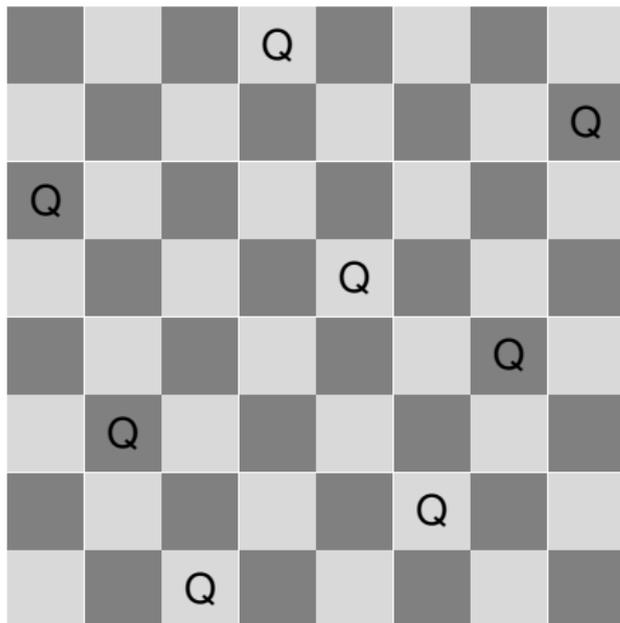
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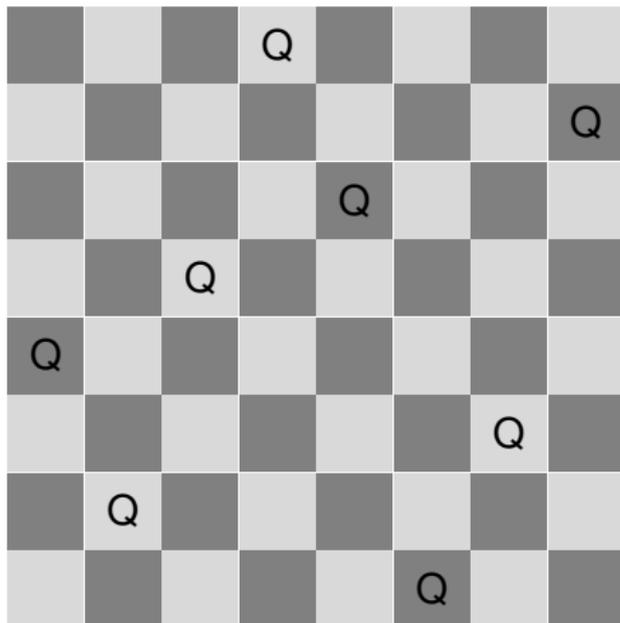
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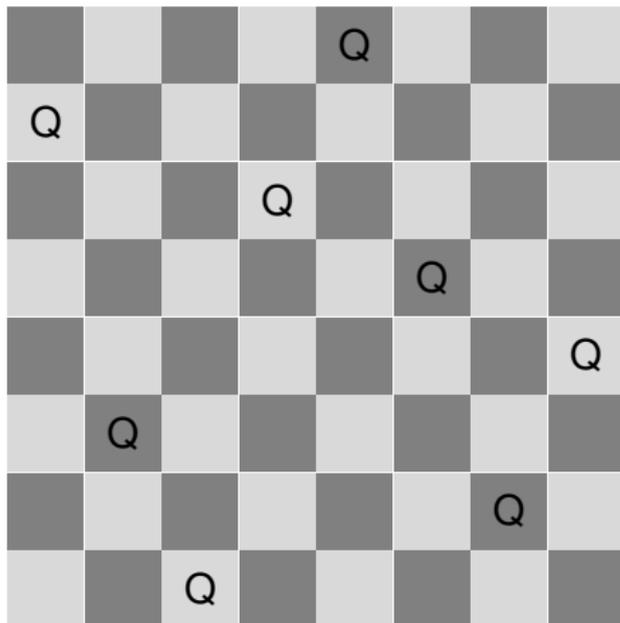
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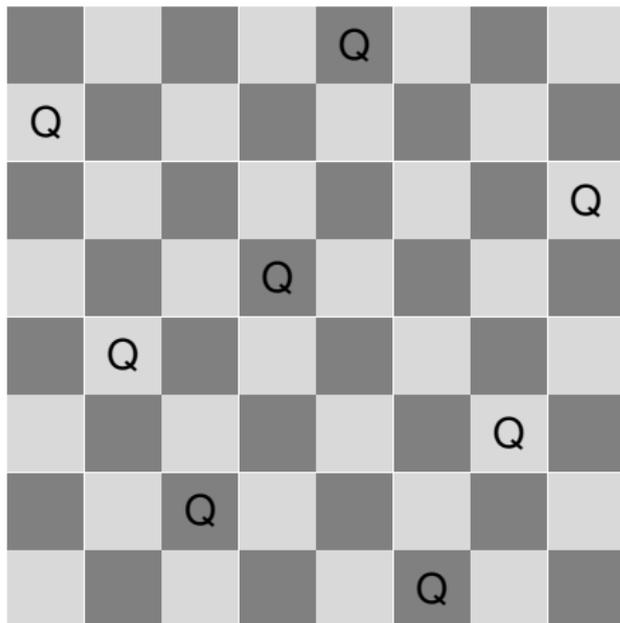
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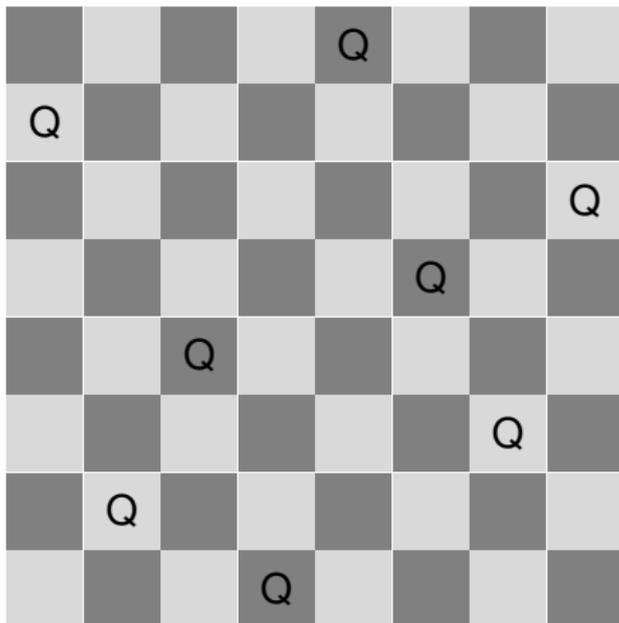
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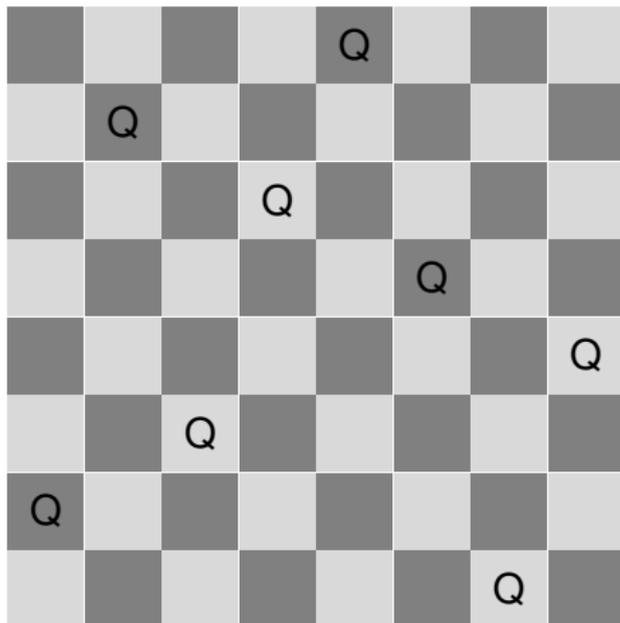
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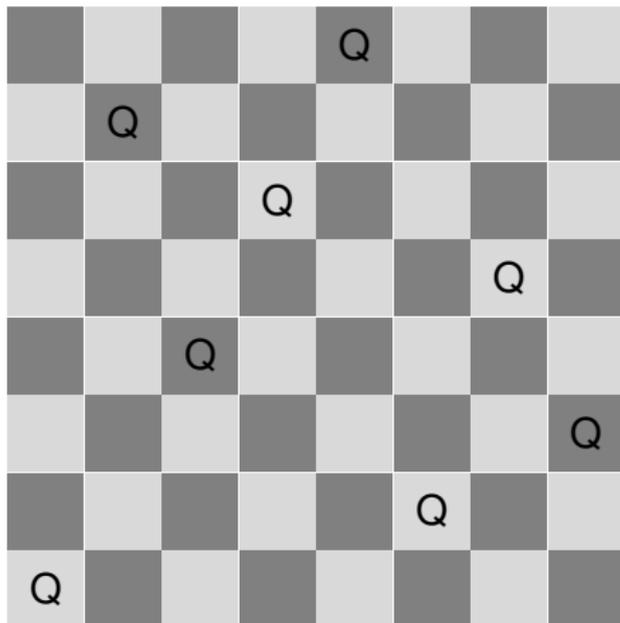
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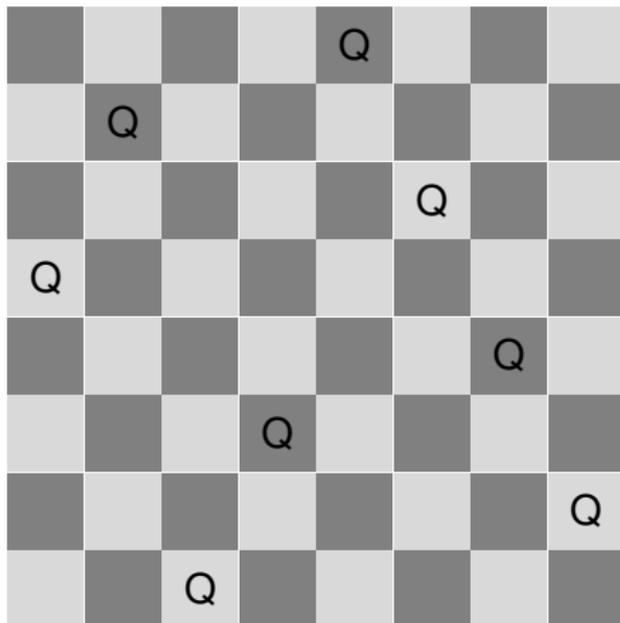
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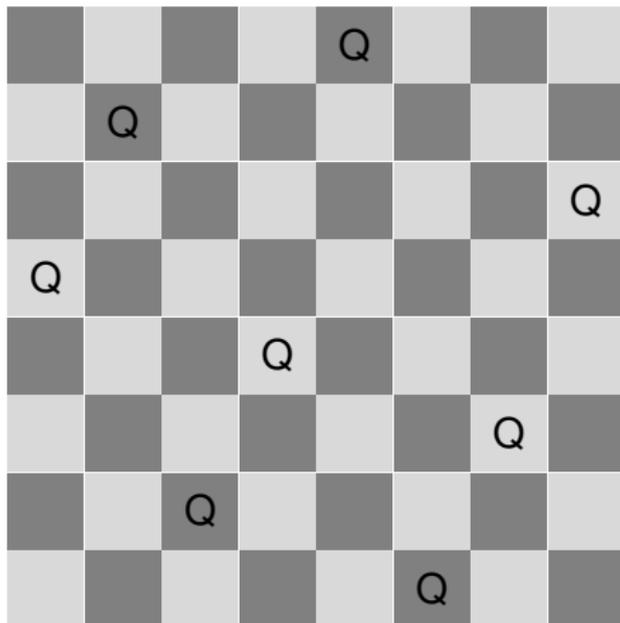
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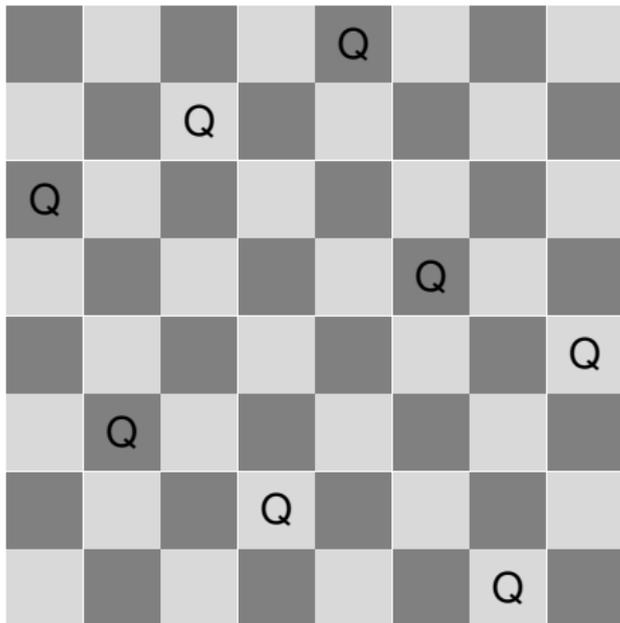
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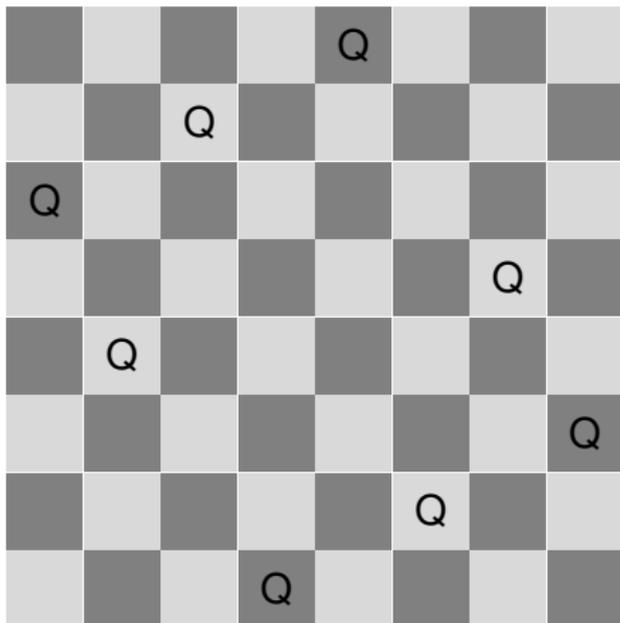
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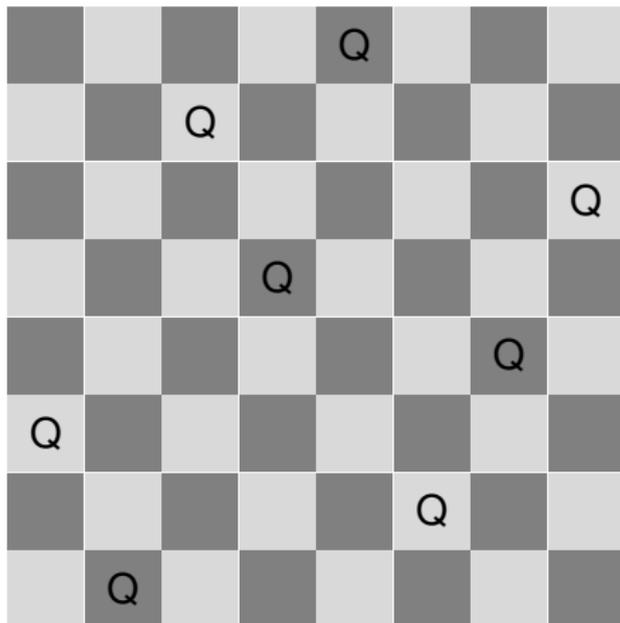
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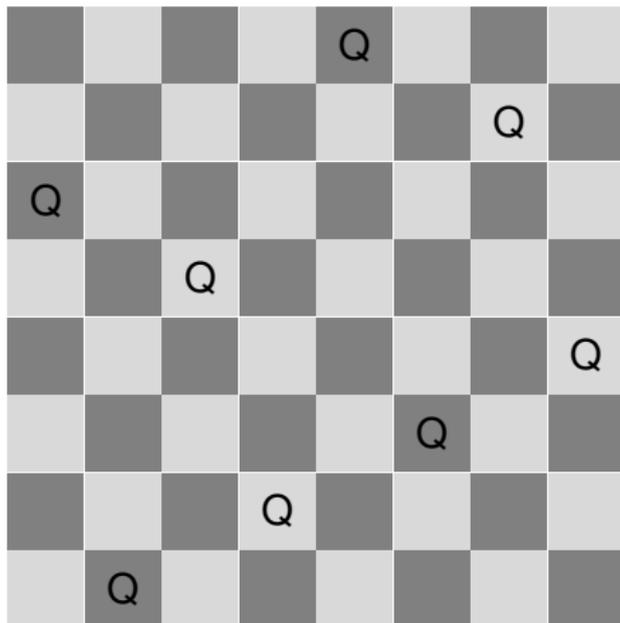
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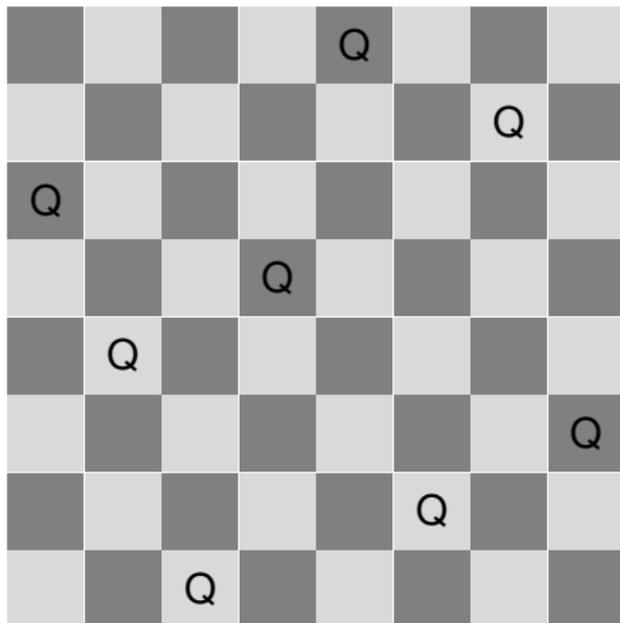
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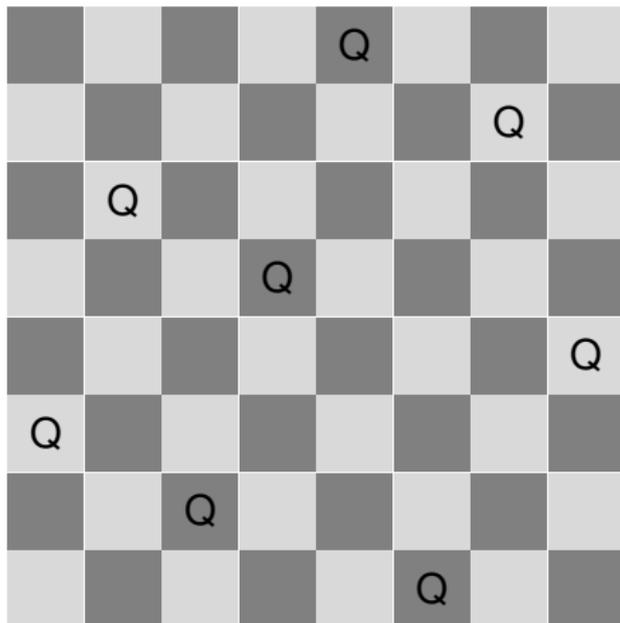
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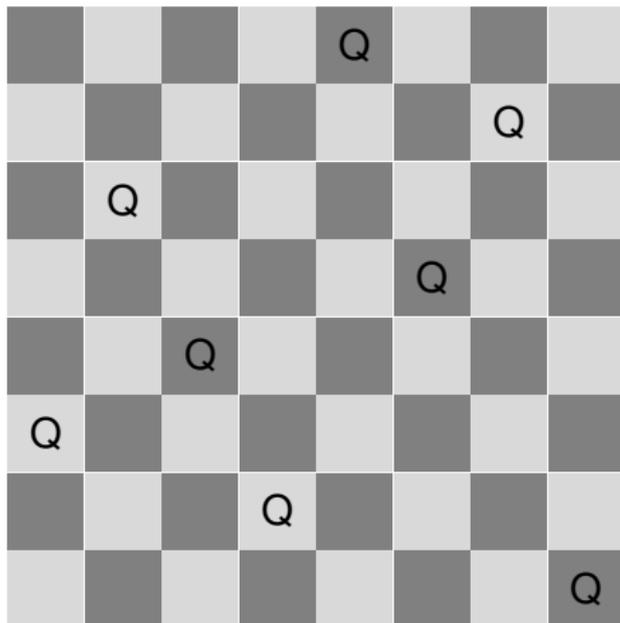
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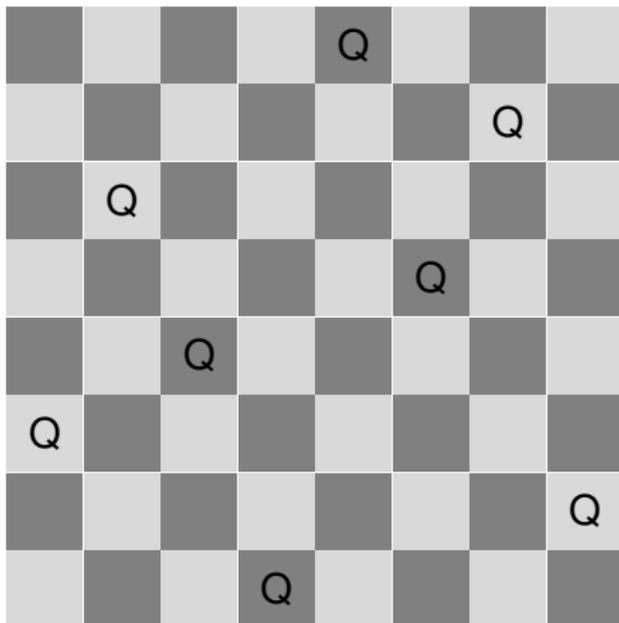
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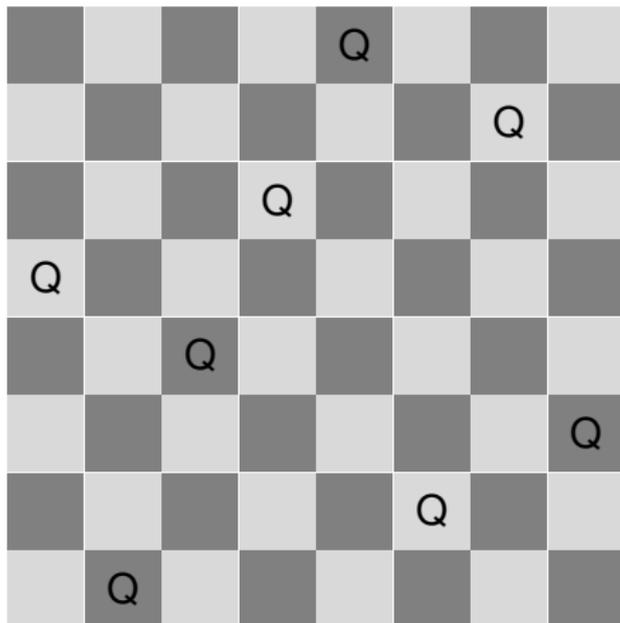
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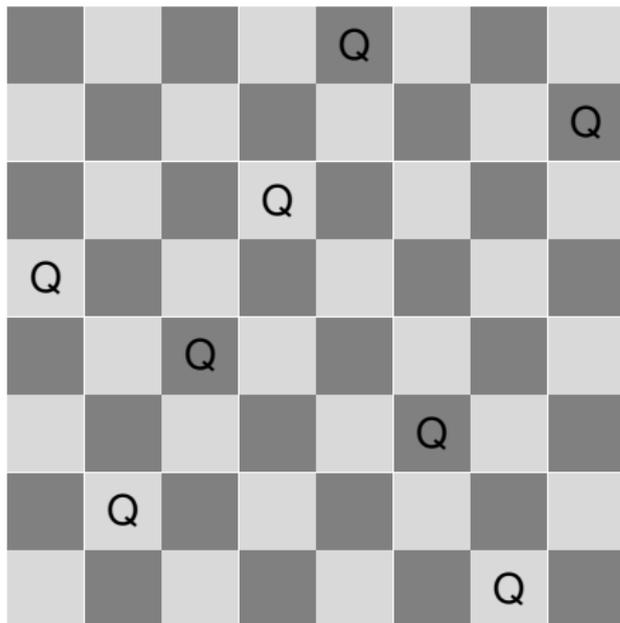
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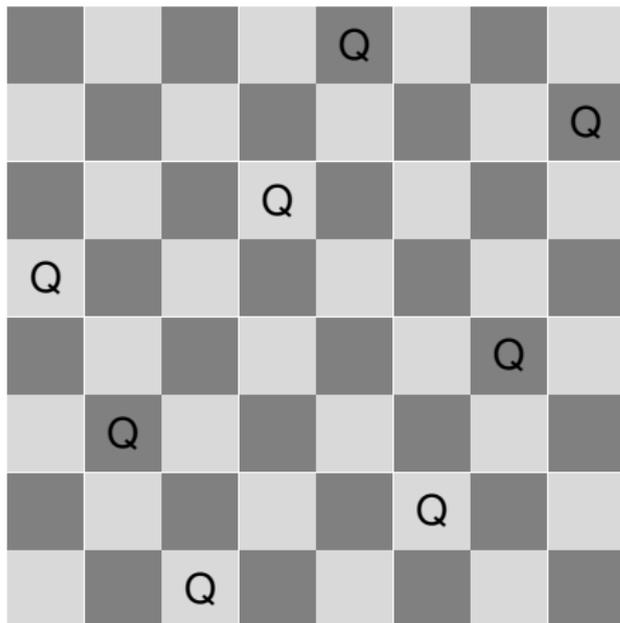
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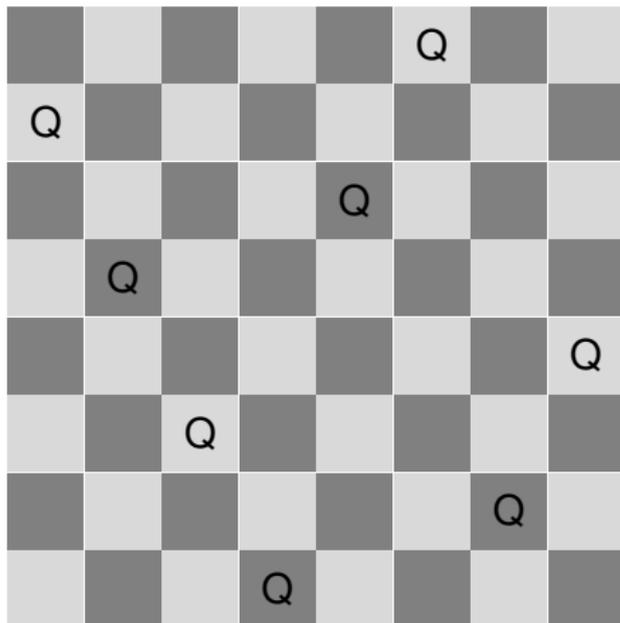
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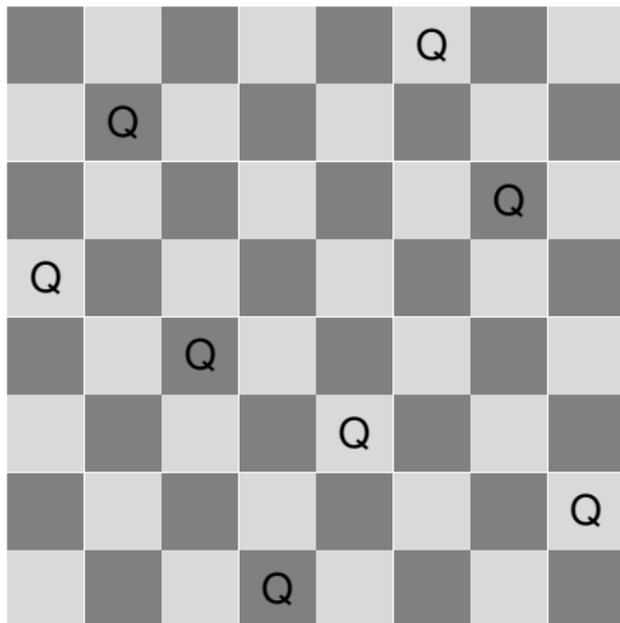
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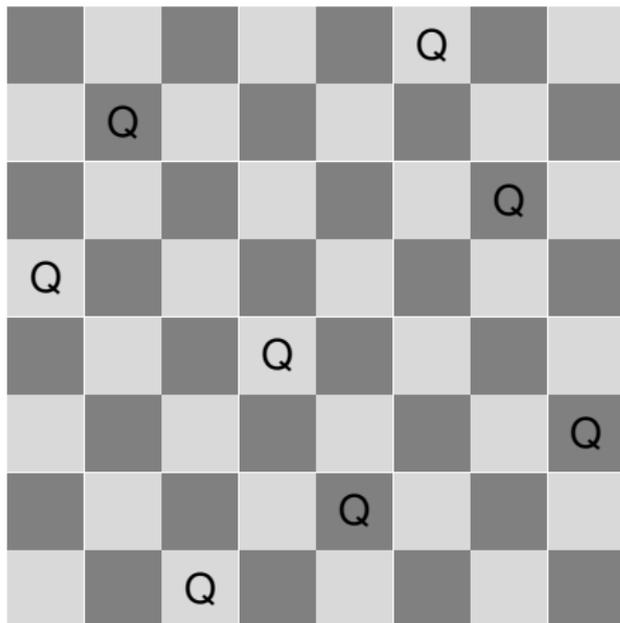
Ocho reinas, todas las soluciones



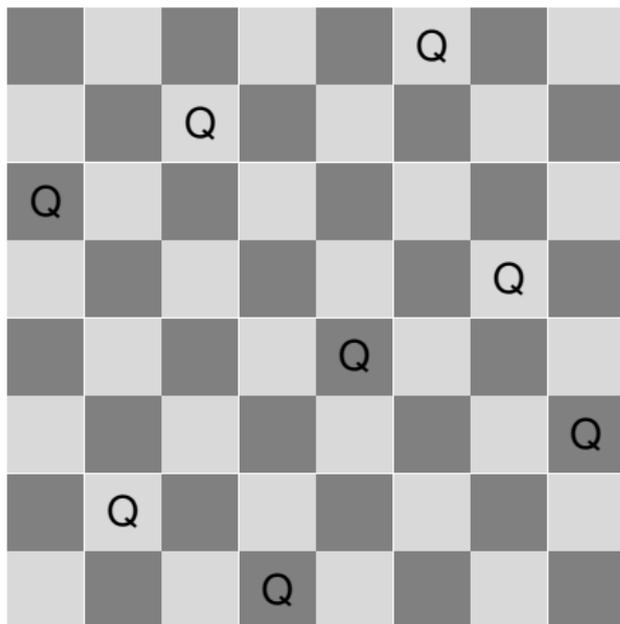
Ocho reinas, todas las soluciones



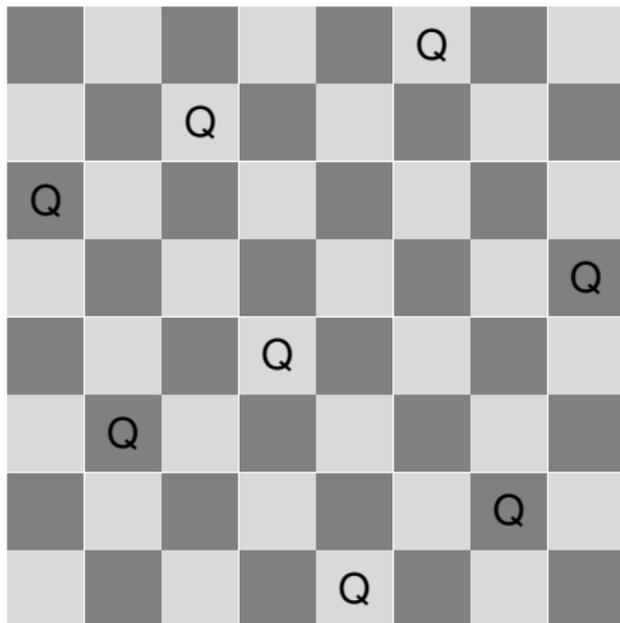
Ocho reinas, todas las soluciones



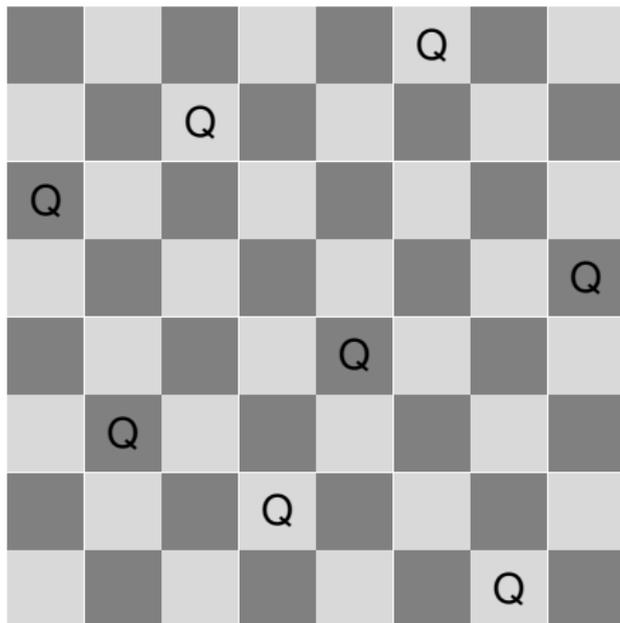
Ocho reinas, todas las soluciones



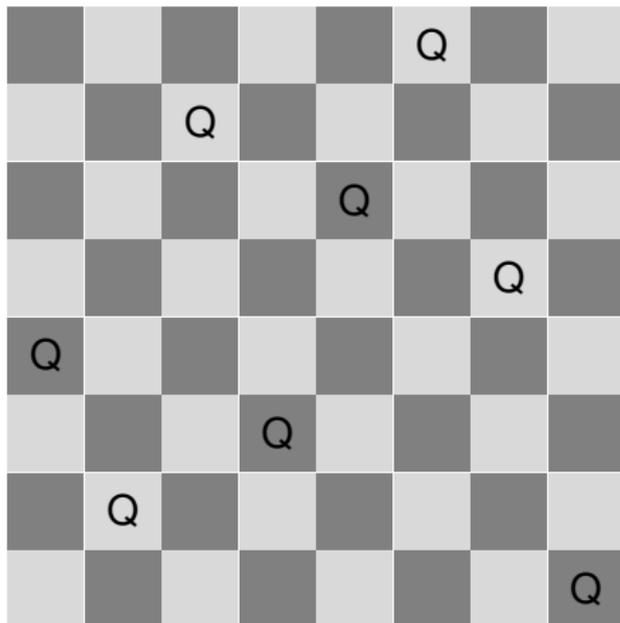
Ocho reinas, todas las soluciones



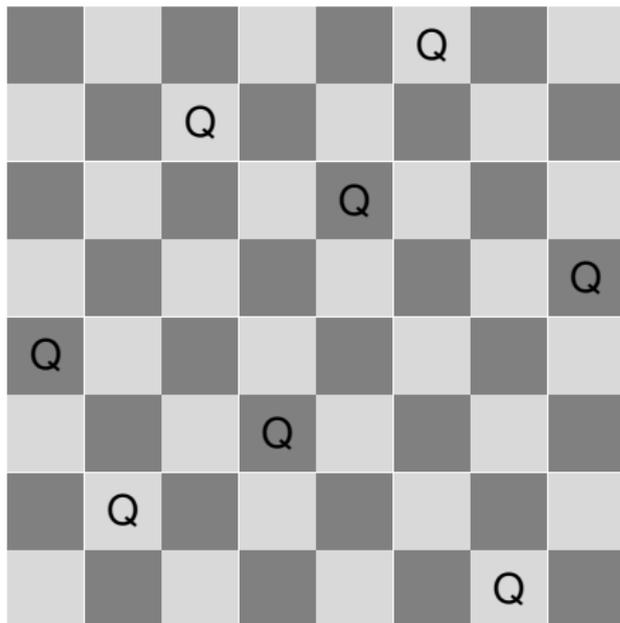
Ocho reinas, todas las soluciones



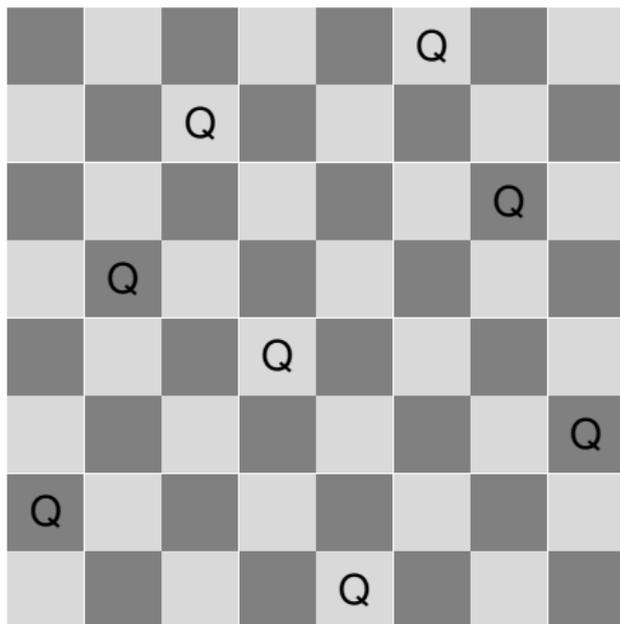
Ocho reinas, todas las soluciones



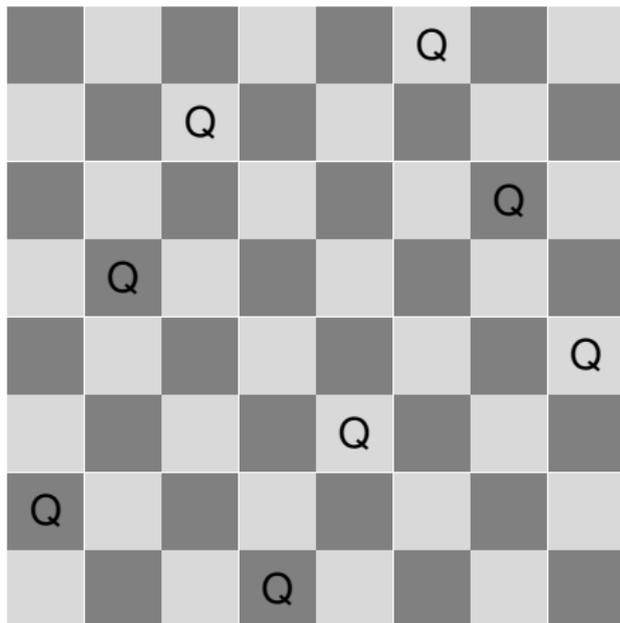
Ocho reinas, todas las soluciones



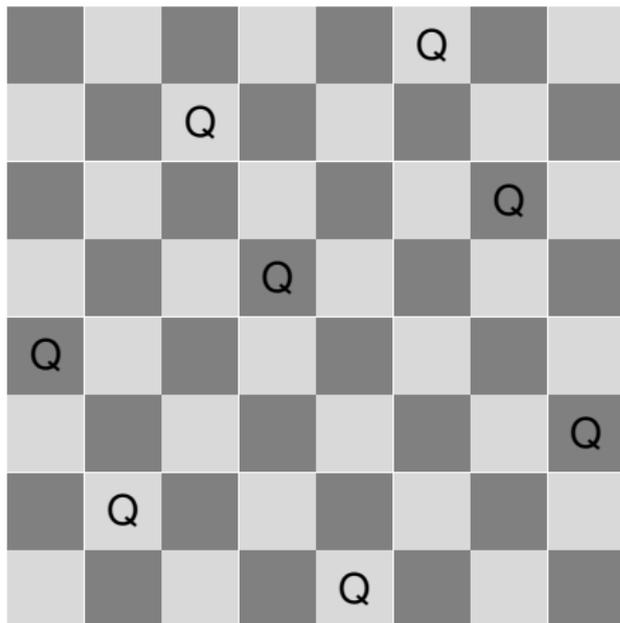
Ocho reinas, todas las soluciones



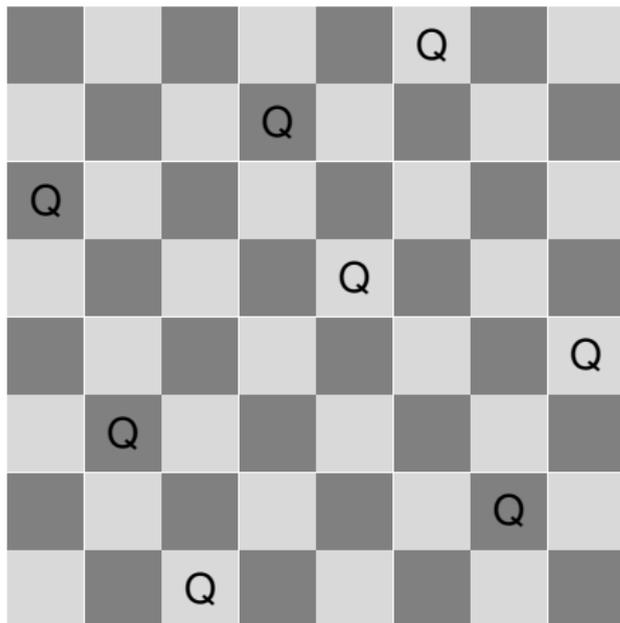
Ocho reinas, todas las soluciones



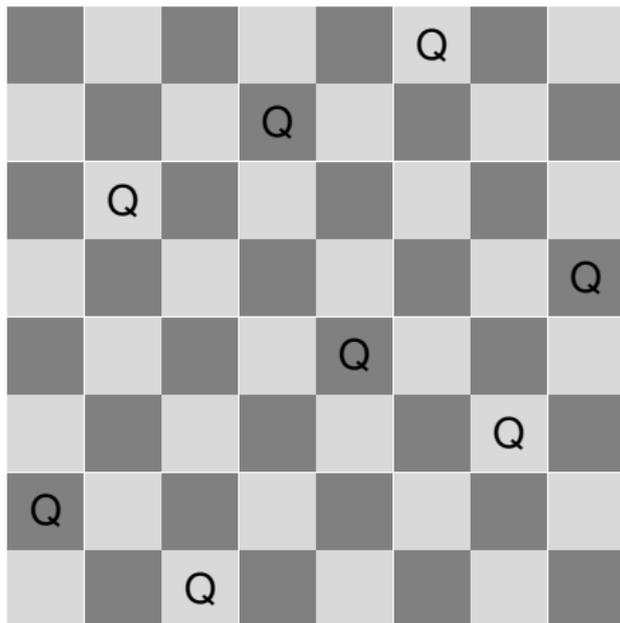
Ocho reinas, todas las soluciones



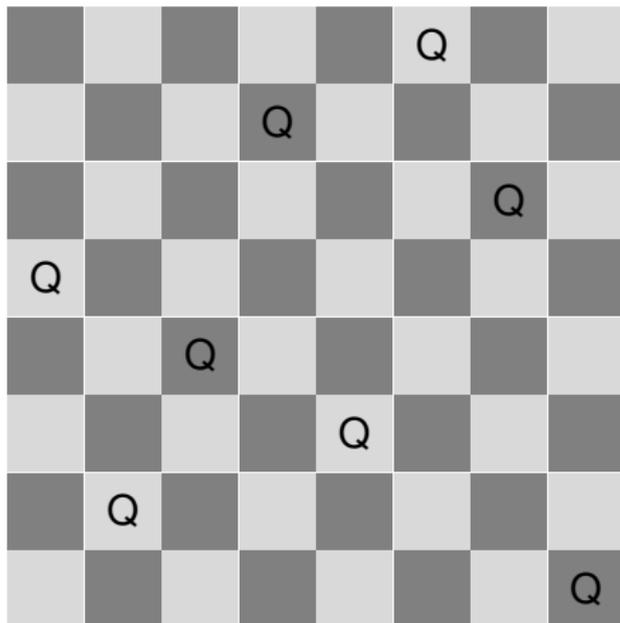
Ocho reinas, todas las soluciones



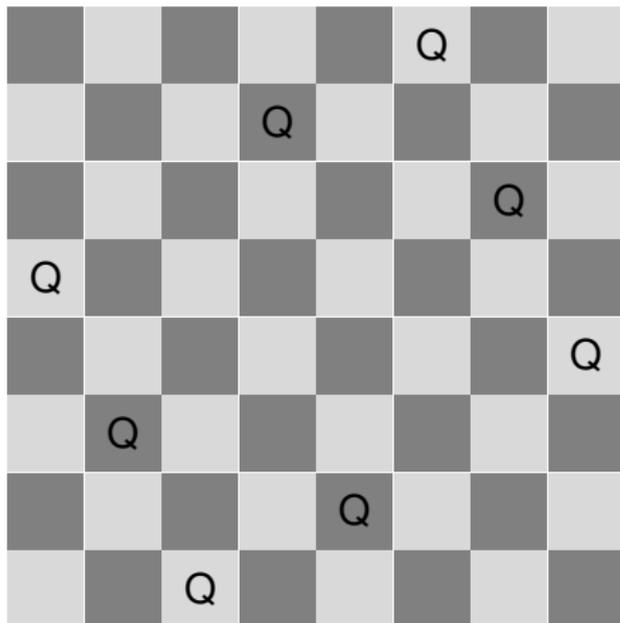
Ocho reinas, todas las soluciones



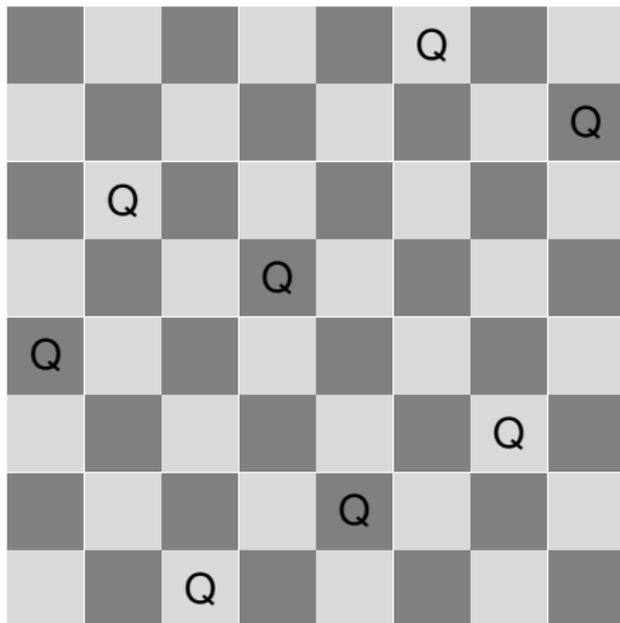
Ocho reinas, todas las soluciones



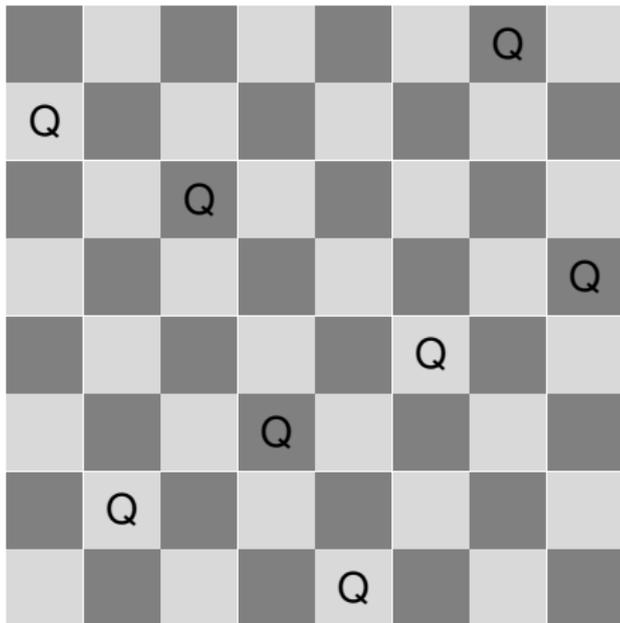
Ocho reinas, todas las soluciones



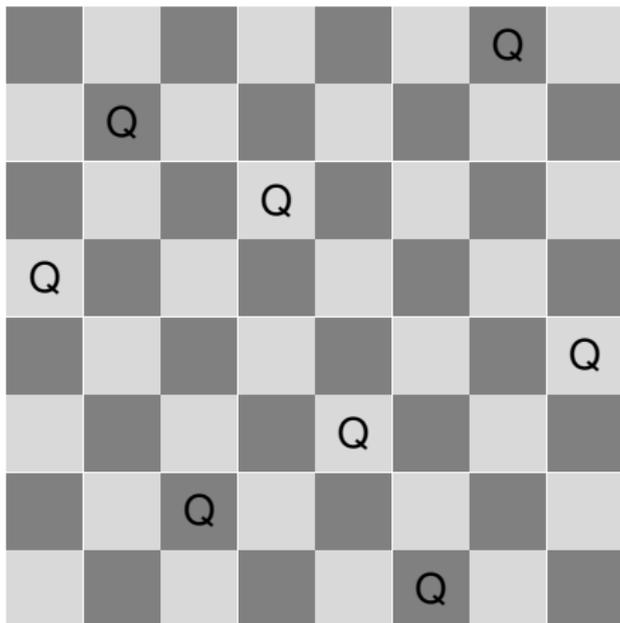
Ocho reinas, todas las soluciones



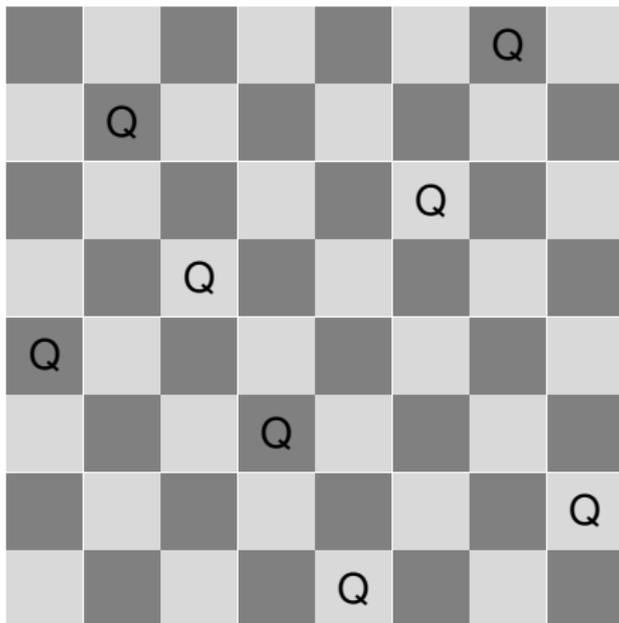
Ocho reinas, todas las soluciones



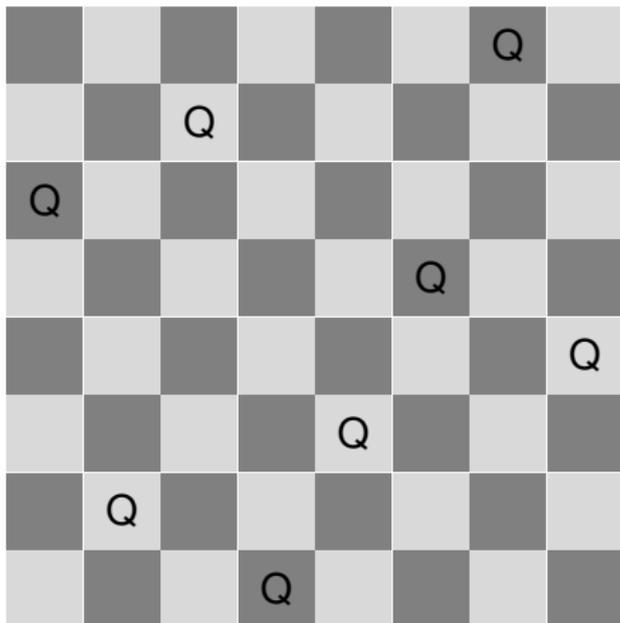
Ocho reinas, todas las soluciones



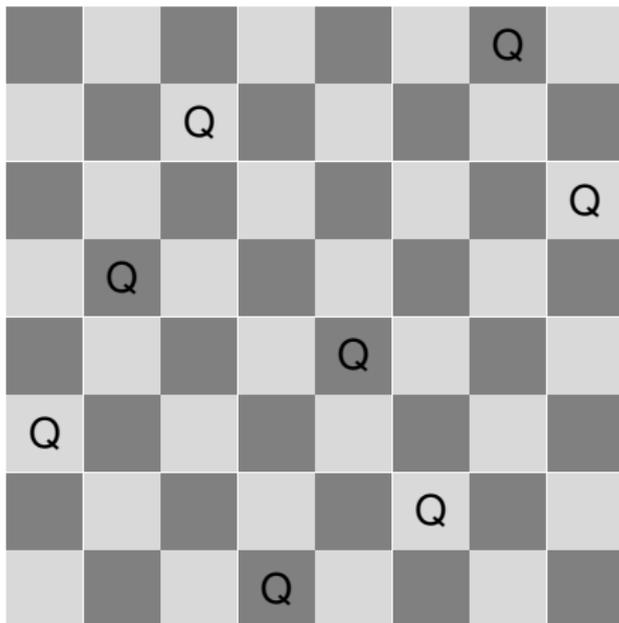
Ocho reinas, todas las soluciones



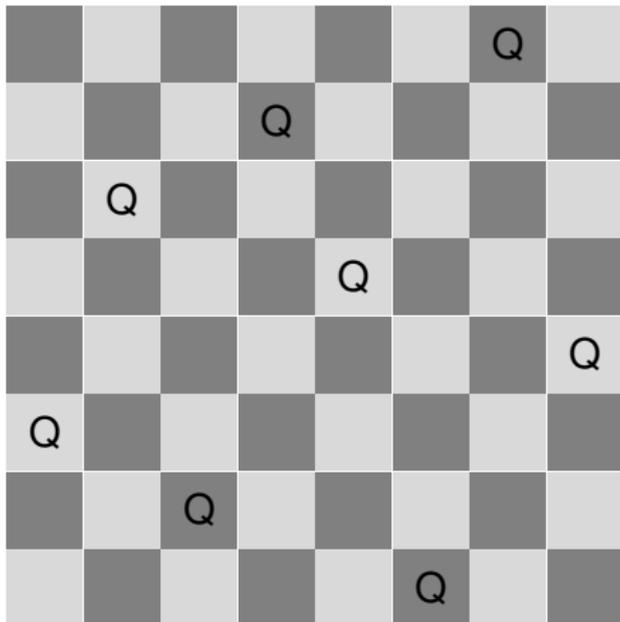
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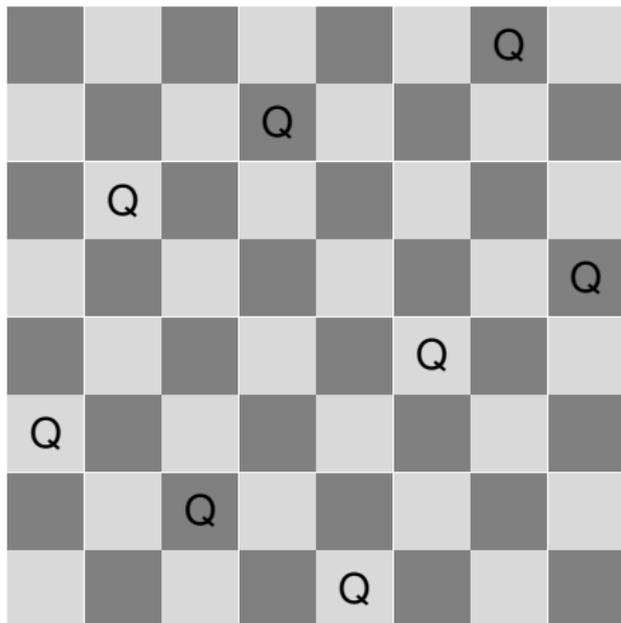
Ocho reinas, todas las soluciones



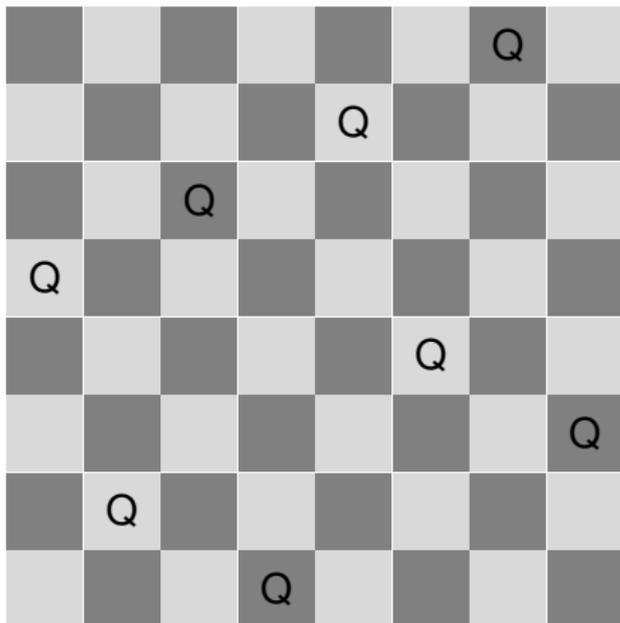
Ocho reinas, todas las soluciones



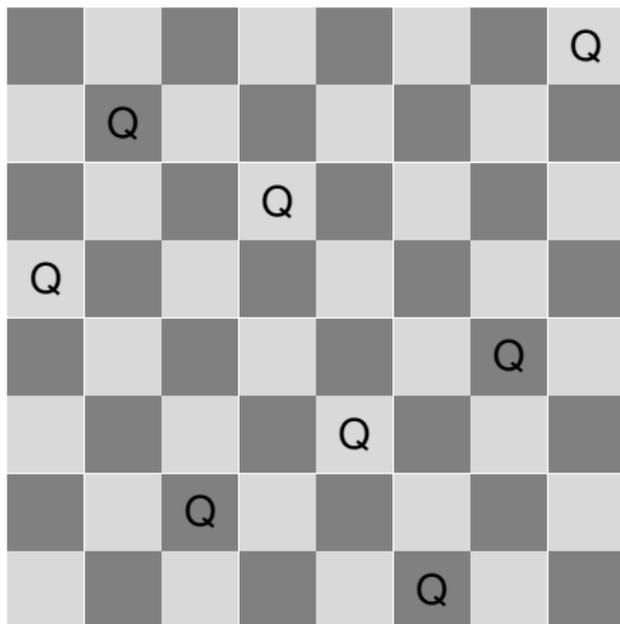
Ocho reinas, todas las soluciones



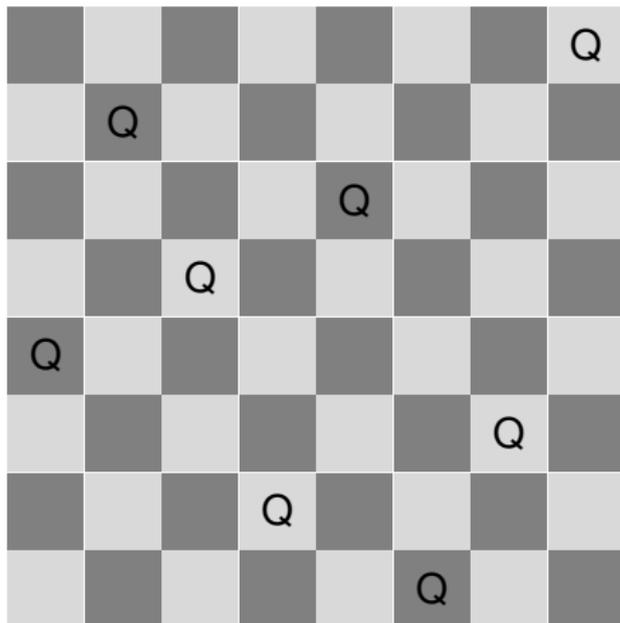
Ocho reinas, todas las soluciones



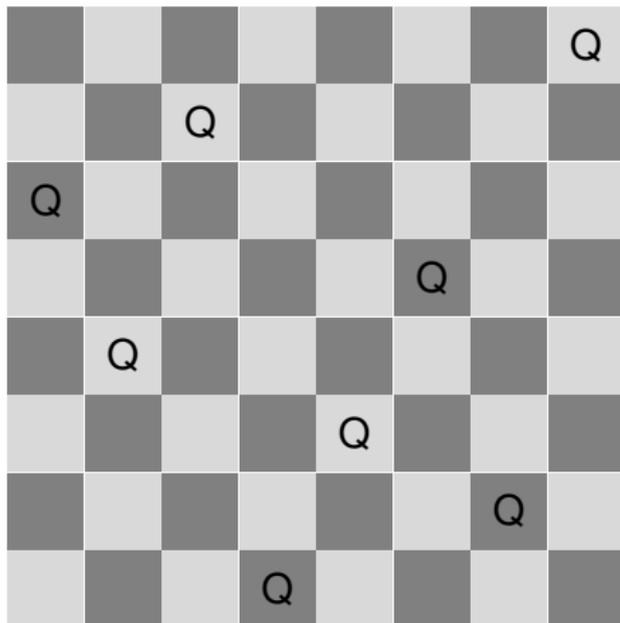
Ocho reinas, todas las soluciones



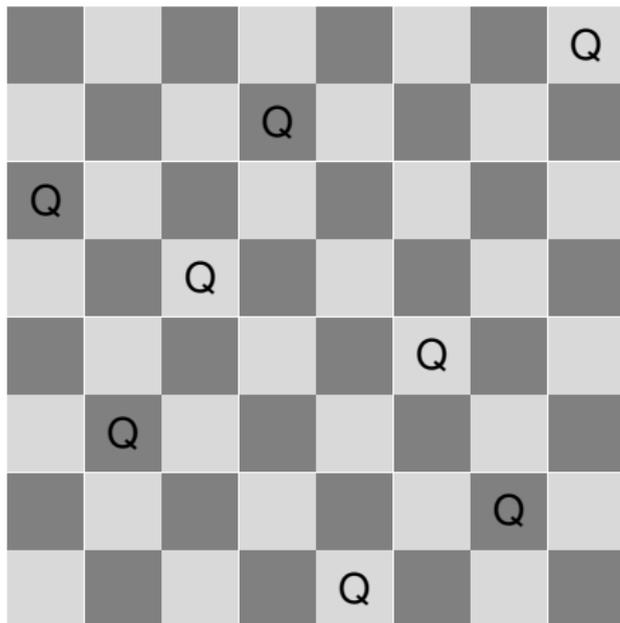
Ocho reinas, todas las soluciones



Ocho reinas, todas las soluciones



Ocho reinas, todas las soluciones



Ocho reinas, un algoritmo optimizado

El algoritmo

```

proc or_4(in sol, bajadas, subidas: list of nat, in/out r: nat)
    {calcula el número de maneras de extender sol}
    {hasta ubicar en total 8 reinas sin que se amenacen}
    {bajadas y subidas son las diagonales ya amenazadas}
if |sol| = 8 then r:= r+1 fi
else i:= |sol|+1
    for j:= 1 to 8 do
        if j ∉ sol ∧ bajada(i,j) ∉ bajadas ∧ subida(i,j) ∉ subidas
        then or_4(sol ◁ j, bajadas ◁ bajada(i,j), subidas ◁ subida(i,j), r)
        fi
    od
fi
end

```

Ocho reinas, un algoritmo optimizado

El algoritmo principal

```
fun ocho_reinas_4() ret r: nat  
  r:= 0  
  or_4([ ], [ ], [ ], r)  
end
```

Sobre bajadas y subidas

Observar que

- Todas las celdas de una bajada tienen en común que la diferencia entre la fila y la columna dan el mismo resultado.
- Todas las celdas de una subida tienen en común que la suma entre la fila y la columna dan el mismo resultado.

Esto sugiere la siguiente idea.

Ocho reinas, un algoritmo optimizado

Algoritmos auxiliares

```
fun bajada(i,j:nat) ret r: nat  
  r:= i-j+7  
end  
fun subida(i,j:nat) ret r: nat  
  r:= i+j  
end
```

Ocho reinas, un algoritmo mejor

El grafo implícito

Ahora resulta más complicado explicitar el grafo implícito: V es el conjunto de listas $[p_1, \dots, p_n] \in \{1, \dots, 8\}^*$ tales que para todo $i \neq j$ las siguientes condiciones se cumplen:

- 1 $p_i \neq p_j$, es decir que no se repiten columnas,
- 2 $p_i - i \neq p_j - j$, es decir que no se repiten bajadas, y
- 3 $p_i + i \neq p_j + j$, es decir que no se repiten subidas.

Las aristas se establecen como en los intentos anteriores.